The Capacity of Wireless Ad Hoc Networks Using Directional Antennas

Pan Li, *Member*, *IEEE*, Chi Zhang, *Student Member*, *IEEE*, and Yuguang Fang, *Fellow*, *IEEE*

Abstract—Considering a disk of unit area with n nodes, we investigate the capacity of wireless networks using directional antennas. First, we study the throughput capacity of random directional networks with multihop relay schemes, and find that the capacity gain compared to random omnidirectional networks is $O(\log n)$, which is tighter than previous results. We also show that using directional antennas can significantly reduce power consumption in the networks. Second, for the first time, we explore the throughput capacity of random directional networks with one-hop relay schemes. Interestingly and against our intuition, we find that one-hop instead of multihop delivery schemes can make random directional networks scale. Third, we investigate the trade-offs between transmission range and throughput in random directional networks and show that using larger transmission range can result in higher throughput. Finally, we present a lower bound on the transport capacity of arbitrary directional networks, and find that without side lobe directional antenna gain, arbitrary directional networks can also scale.

Index Terms—Wireless ad hoc networks, throughput capacity, transport capacity, directional antenna, power consumption.

1 Introduction

In wireless networks, users communicate with each other over a common wireless channel. Due to the wireless broadcast nature, the capacity of wireless networks is constrained by the bandwidth and the node densities. Gupta and Kumar [8] show that when using omnidirectional antennas, a random network with n nodes can provide a per-node throughput at $\Theta(W/\sqrt{n \log n})$ bits per second, where W is the channel capacity and $\Theta(x)$ indicates a function on the same order of x. They also show that even under optimal circumstances, the capacity is only $\Theta(W/\sqrt{n})$ bit-meters per second for each node. In other words, wireless networks cannot scale when using omnidirectional antennas.

Inspired by this seminal paper, many researchers have investigated the capacity issue for wireless networks under various constraints. Li et al. [10] examine the capacity of wireless ad hoc networks by using simulations based on IEEE 802.11 protocols. Agarwal and Kumar [1] revisit the capacity problem and derive some improved capacity bounds for wireless networks. All these works still show that wireless networks using omnidirectional antennas cannot scale. Later, Ozgur et al. study the throughput capacity of wireless ad hoc networks in [24] and [25]. Their results reveal that by intelligent node cooperation and distributed MIMO communication, wireless networks can scale linearly with the number of nodes. Besides, some papers like [2], [4], [6], [12], [18], [21], [22] propose to use mobility to increase network throughput. A constant per-node throughput can be obtained at the cost of very large network delay. Furthermore, some other works such as [9], [11], [17], [19], [20], [26], [34] study the capacity of hybrid wireless networks where base stations are placed to help improve the network performance. They show that hybrid wireless networks could also provide a constant per-node throughput at the cost of high infrastructure investment. In this paper, we focus on the capacity of pure wireless ad hoc networks, which we call wireless networks for simplicity.

Recently, directional antennas have emerged as a promising technology due to the higher spatial reuse ratio, the improved communication distance, and the reduced interference. Li et al. [15], [16] study the connectivity problem in wireless networks using directional antennas. Some other works, such as [3], [5], [13], [14], [28], [30], explore the MAC protocol design with the use of directional antennas attempting to improve the network throughput. In addition, Yap et al. [31] analyze the throughput of several contention-based MAC protocols when simple directional antenna models are employed. Some performance gains have been shown over the omnidirectional antenna case. However, there is still one open question: how well on earth can wireless networks using directional antennas do?

Using the max-flow/min-cut theorem in flow networks, Peraki and Servetto [27] show that random networks with directional antennas can achieve an increase of $\Theta(\log^2(n))$ in maximum stable throughput compared to random networks with omnidirectional antennas. However, as the authors point out in the paper, the problem considered there has certain restrictions to the general one considered in [8] and also in this paper. Yi et al. [32], [33] also study the capacity improvement of ad hoc networks using directional antennas. For arbitrary networks, by using a simple directional antenna model without the side lobe gain, they show that the capacity gain is $\sqrt{2\pi/\alpha}$ when using directional transmission and omnireception, $\sqrt{2\pi/\beta}$ when using omnitransmission and directional reception, and

- *P. Li is with the Department of Electrical and Computer Engineering, Mississippi State University, 229 Simrall Engineering Building, Mississippi State, MS 39762-9571. E-mail: li@ece.msstate.edu.*
- *C. Zhang is with the Department of Electrical and Computer Engineering, University of Florida, Gainesville, FL 32611. E-mail: zhangchi@ufl.edu.*
- *Y. Fang is with the Department of Electrical and Computer Engineering, University of Florida, Gainesville, FL 32611, and is also a Changjiang Scholar Chair Professor with the National Key Laboratory of Integrated Services Networks, Xidian University, Xi'an 710071, China. E-mail: fang@ece.ufl.edu.*

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$2\pi/\sqrt{\alpha\beta}$ when using both directional transmission and directional reception (DTDR), respectively. For random networks, they show that, by ignoring the side lobe gain, the capacity gain can be $2\pi/\alpha$, $2\pi/\beta$, and $4\pi^2/\alpha\beta$, for directional transmission and omnireception, omnitransmission and directional reception, and directional transmission and directional reception, respectively, where α and β are the beamwidths of a transmitter and a receiver, respectively. However, no upper bound on the capacity gain is given. Besides, the analysis in [32] and [33] is still based on the same Protocol Model proposed in [8], while we contend that a different interference model should be used to characterize the interference in directional wireless networks. Also, note that Peraki and Servetto [27] and Yi et al. [32], [33] study the throughput capacity only when multihop relay schemes are used in the network.

In this paper, we revisit the capacity problem in wireless networks using directional antennas. We employ a more practical directional antenna model than in [27], [32], and [33], with the beam number N, the main lobe gain G_m, and the side lobe gain G_s. According to the transmission and the reception schemes with the use of directional antennas, we classify wireless networks into four categories, which are Directional Transmission and Directional Reception networks, Omnidirectional Transmission and Directional Reception (OTDR) networks, Directional Transmission and Omnidirectional Reception (DTOR) networks, and Omnidirectional Transmission and Omnidirectional Reception (OTOR) networks. Moreover, we propose a new Directional Protocol Model based on the Physical Model for the capacity analysis. Our contribution is fourfold stated as follows:

First, we investigate the throughput capacity in random directional networks using multihop relay schemes, and show that the capacity gain compared to random omnidirectional networks (OTOR networks) is $O(\log n)$ when the side lobe gain of directional antennas is very small, i.e., when $G_s/{G_m}^{1/\alpha} = o(1)$ where $\alpha > 2$ is the path loss exponent, and is on a constant order $G_s/G_m = \Theta(1)$. Note that these results are much tighter than those in [27], [32], and [33] as we introduced before. Besides, previous works do not take power consumption into consideration, while here, we find that the use of directional antenna can significantly reduce power consumption in the network. In addition, we also show that a lower bound on the per-node throughput capacity in random DTDR, OTDR, and DTOR networks is the same as that in random OTOR networks, i.e., $\Omega(1/\sqrt{n\log n})$.

Second, we explore the throughput capacity in random directional networks using one-hop delivery schemes for the first time. Let $a = (G_s/G_m)^{1/\alpha} (0 \le a \le 1)$. We find that the per-node throughput capacity in random OTOR, DTDR, OTDR, and DTOR networks scales as $\Theta(1/n)$, $\Theta(N^2/n)$, $\Theta(N/n)$, and $\Theta(N/n)$, respectively, when $a = o(1/\sqrt{N})$, and as $\Theta(1/n)$, $\Theta(a^{-4}/n)$, $\Theta(a^{-2}/n)$, and $\Theta(a^{-2}/n)$, respectively, when $a = \Omega(1/\sqrt{N})$. Let $a = N^x$ where $x \le 0$. So, random DTDR, OTDR, and DTOR networks using one-hop delivery schemes can scale if the beam number N scales as $\sqrt{n}$, n, and n, respectively, when $x < -\frac{1}{2}$, or as $n^{-\frac{1}{4x}}$, $n^{-\frac{1}{2x}}$, and $n^{-\frac{1}{2x}}$, respectively, when $-\frac{1}{2} \le x < 0$. Moreover, we show that with one-hop delivery schemes, the transmission power for each node in the network can be upper bounded by a constant.

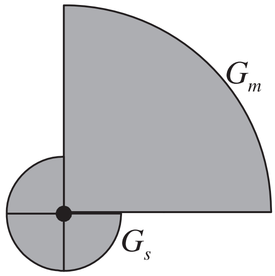

Fig. 1. Our directional antenna model with four directions.

Third, noticing that one-hop delivery schemes can provide higher throughput than multihop schemes, we investigate the trade-offs between transmission range and throughput in random directional networks. We show that when directional antennas are used, using larger transmission range can lead to higher throughput, which is very different from random omnidirectional networks. We also find the conditions on directional antenna design for the transmission power to be upper bounded by a constant.

Fourth, we present a lower bound on the transport capacity in arbitrary directional networks, and find that without side lobe directional antenna gain, the per-node transport capacity scales as $\Omega(N/\sqrt{n})$, $\Omega(\sqrt{N/n})$, and $\Omega(\sqrt{N/n})$, respectively, in arbitrary DTDR, OTDR, and DTOR networks. Note that the capacity for arbitrary directional networks in [32] and [33] is obtained using the approach in [8] for deriving an upper bound on the capacity. So, the results there cannot serve as lower bounds on the capacity.

The rest of this paper is organized as follows: We introduce our models and definitions in Section 2. Sections 3 and 4 show a lower bound and an upper bound on the throughput capacity of random directional networks using multihop relay schemes, respectively. Sections 5 and 6 present a lower bound and an upper bound on the throughput capacity of random directional networks employing one-hop delivery schemes, respectively. Section 7 shows the trade-offs between transmission range and throughput in random directional networks. Section 8 presents a lower bound on the transport capacity of arbitrary directional networks. We finally conclude this paper in Section 9.

2 Models and Definitions

2.1 Directional Antenna Model

As in [16], in this study, we use the switched beam antenna system which consists of several highly directive, fixed, predefined beams, and each transmission uses only one of the beams. We assume that every antenna has $N (N > 1)$ beams exclusively and collectively covering all directions, and that the main lobe antenna gain G_m and the side lobe antenna gain G_s are constant in the transmission direction and nontransmission directions, respectively. One such directional antenna with four beam directions is shown in Fig. 1. Note that *N, G_m, and G_s are design parameters rather than simple constants*. Besides, we have $0 \le G_s < 1 < G_m$ when directional antennas work in the directional mode, and $G_s = G_m = 1$ when they work in the omnidirectional mode, respectively.

Let P be the transmission power, and S the surface area of the sphere with center at the transmitter and radius R. As shown in Fig. 2, the surface area A on the sphere for a

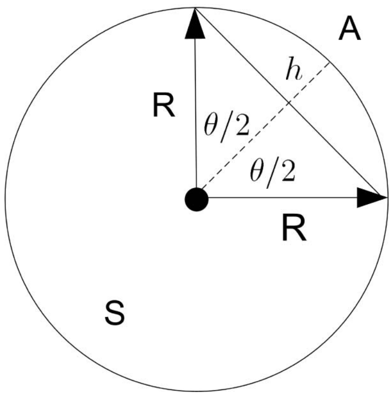

Fig. 2. Illustration for calculating the main and side lobe antenna gain.

beamwidth of θ is $2\pi Rh$, where h is $R(1-\cos\frac{\theta}{2})$. As we have shown in [16], we have

$$G_m \cdot A + G_s \cdot (S - A) = \eta \cdot S, \tag{1}$$

where $A = 2\pi R^2(1-\cos\frac{\theta}{2})$, $S = 4\pi R^2$, and $\eta (0 < \eta \leq 1)$ is the efficiency of the antenna which accounts for losses.

2.2 Power Propagation Model

In this paper, we also employ a general power propagation model to predict the received signal strength [29]:

$$P_r(d) = P_t C \frac{G_t G_r}{d^\alpha}, \tag{2}$$

where P_t and P_r are the transmitted power and the received power, respectively, G_t and G_r are the gain factors for the transmitter's antenna and the receiver's antenna, respectively, C is a constant determined by antenna heights, wavelength, and so on, d is the distance between a transmitter and a receiver, and α is the path loss exponent which is usually bigger than 2.

2.3 Network Model

We consider a dense network with n nodes distributed in a disk of unit area. In dense *random* networks, the n nodes are randomly distributed, i.e., independently and uniformly distributed. We follow the process in [6] to choose random sender-receiver pairs so that each node is a source node for one flow and a destination node for at most $O(1)$ flows. We assume that all nodes have a common transmission range $r(n)$ and no power control is employed. Besides, all nodes use switched beam directional antennas which have the same antenna pattern, and randomly beamform in one of the N directions with equal probability. While in dense *arbitrary* networks, the n nodes are arbitrarily placed. The destinations of source nodes and the transmission power are arbitrarily chosen. All nodes use the same switched beam directional antennas which can beamform to an arbitrary direction.

2.4 Interference Model

2.4.1 The Physical Model

Let $\mathbb{T}$ be the subset of nodes simultaneously transmitting at some time instant and P the common transmission power level chosen by the nodes in the network. Then, the transmission from a node $T_i \in \mathbb{T}$ is successfully received by a node R_i if

$$\frac{PC\frac{G_t G_r}{|T_i - R_i|^\alpha}}{N_0 + \sum_{T_k \in \mathbb{T}, k \neq i} PC\frac{G_t G_r}{|T_k - R_i|^\alpha}} \geq \beta, \tag{3}$$

where T_i and R_i also denote nodes' positions, N_0 is the ambient noise power level at the receiver, and β is the minimum signal-to-interference plus noise ratio (SINR) for successful receptions. Note that in most cases, we have $\beta > 1$.

2.4.2 The Protocol Model

Suppose a node T_i transmits to a node R_i. Then, in order for this transmission to be successful, two conditions need to be satisfied [8]:

- The distance between T_i and R_i is no more than $r(n)$, i.e., $|T_i - R_i| \leq r(n)$.
- The positions of every other transmitters T_j simultaneously transmitting should satisfy:

$$|T_j - R_i| \geq (1+\Delta) r(n). \tag{4}$$

  The quantity $\Delta > 0$ models situations where a guard zone is specified by the protocol to prevent a neighboring node from transmitting at the same time. It also allows for imprecision in the achieved range of transmissions.

Note that this Protocol Model will be used in random OTOR networks only. For the analysis in random DTDR, DTOR, and OTDR networks, we will use a new Directional Protocol Model proposed based on the Physical Model, which will be introduced later.

2.5 Capacity

As defined in the usual way, the time average of the number of bits that can be transmitted by each node to its destination is called the *per-node throughput*, and the total number of bits per second that can be transmitted by all nodes to their destinations is called the *network throughput*, or *throughput of the network*. In this paper, we assume that each transmitter intends to send $\lambda(n)$ bits per second to its destination node.

Besides, as defined in [8], we say a network transports 1 *bit-meter* when 1 bit has been transported a distance of 1 meter from a source toward its destination. This sum of products of bits and the distances over which they are carried is called *Transport Capacity*.

3 A Lower Bound on the Throughput Capacity of Random Networks—Multihop Relay

In this section, we derive a lower bound on the throughput capacity of random networks using directional antennas when multihop relay schemes are employed. Recall that we assume all nodes randomly beamform in one of the N directions with equal probability. As we will see later, a lower bound on the network throughput capacity can be achieved by scheduling the transmission of the nodes in the interference zone, one by one. Thus, it is reasonable for us to assume that a transmitter and its corresponding receiver can beamform to each other at the time when it is their turn to carry out the transmission.

3.1 Random DTDR Networks

In this section, we derive a lower bound on the throughput capacity of random DTDR networks. Consider the setting on a planar disk. We first introduce some definitions and results we will use. c_is are used to denote deterministic constants independent of n.

Voronoi tessellation. Given a set of n points in a plane, Voronoi tessellation divides the domain into a set of polygonal regions, the boundaries of which are the perpendicular bisectors of the lines joining the points [23].

Lemma 1. *For every $\varepsilon > 0$, there is a Voronoi tessellation with the property that every Voronoi cell contains a disk of radius ε and is contained in a disk of radius 2ε [8].*

Then, for n nodes, we can construct a Voronoi tessellation V_n for which:

V1. Every Voronoi cell contains a disk of area $100 \log n/n$.
V2. Every Voronoi cell is contained in a disk of radius $2\rho(n)$, where $\rho(n) := \text{the radius of a disk of area} \frac{100 \log n}{n}$.

We refer to each Voronoi cell $V \in V_n$ as simply a cell.

Adjacent cells. Two cells are adjacent if they share a common point (every cell is a closed set).

Since a transmitter and its intended receiver can beamform to each other to carry out the transmission, we choose the transmission range, denoted by $r_{mm}(n)$, to be $r_{mm}(n) = r(n) = 8\rho(n)$, which allows direct communication within a cell and between adjacent cells.

Interfering cells. A cell is an interfering cell of another one if two transmissions from the nodes in these two cells, respectively, interfere with each other.

Remember that the Protocol Model in [8] was proposed for omnidirectional wireless networks only. Here, in order to characterize the interference in directional wireless networks, we propose a new **Directional Protocol Model** based on the Physical Model.

Now, recall the Physical Model. Suppose a node T_i transmits to a node R_i and they can beamform to each other according to our assumptions. Let $\mathbb{T}^1$, $\mathbb{T}^2$, and $\mathbb{T}^3$ denote three different sets of nodes transmitting at the same time as T_i, where $T_j \in \mathbb{T}^1$ and R_i beamform to each other, either $T_k \in \mathbb{T}^2$ or R_i beamforms to the other node (but not both), and neither $T_l \in \mathbb{T}^3$ nor R_i beamforms to the other node, respectively. Then, from (3), it follows that

$$\frac{P\frac{CG_mG_m}{|T_i-R_i|^\alpha}}{\sum_{T_j\in\mathbb{T}^1} P\frac{CG_mG_m}{|T_j-R_i|^\alpha} + \sum_{T_k\in\mathbb{T}^2} P\frac{CG_mG_s}{|T_k-R_i|^\alpha} + \sum_{T_l\in\mathbb{T}^3} P\frac{CG_sG_s}{|T_l-R_i|^\alpha}} \geq \beta.$$

Thus, we have

$$\begin{cases} |T_j - R_i| \geq \beta^{\frac{1}{\alpha}}|T_i - R_i|, & \text{if } T_j \in \mathbb{T}^1, \\ |T_k - R_i| \geq \beta^{\frac{1}{\alpha}}\big(\frac{G_s}{G_m}\big)^{\frac{1}{\alpha}}|T_i - R_i|, & \text{if } T_k \in \mathbb{T}^2, \\ |T_l - R_i| \geq \beta^{\frac{1}{\alpha}}\big(\frac{G_s}{G_m}\big)^{\frac{2}{\alpha}}|T_i - R_i|, & \text{if } T_l \in \mathbb{T}^3. \end{cases}$$

Choosing $\triangle = \beta^{\frac{1}{\alpha}} - 1$ gives us a Protocol Model based on the Physical Model.

Therefore, we propose a **Directional Protocol Model** stated as follows: suppose a node T_i transmits to a node R_i and $|T_i - R_i| \leq r(n)$. Then, the positions of every other transmitters T_j simultaneously transmitting should satisfy:

$$\begin{cases} |T_j - R_i| \geq (1+\triangle)r(n), & \text{if } E_1, \\ |T_j - R_i| \geq (1+\triangle)r(n) \cdot \big(\frac{G_s}{G_m}\big)^{\frac{1}{\alpha}}, & \text{if } E_2, \\ |T_j - R_i| \geq (1+\triangle)r(n) \cdot \big(\frac{G_s}{G_m}\big)^{\frac{2}{\alpha}}, & \text{if } E_3, \end{cases}$$

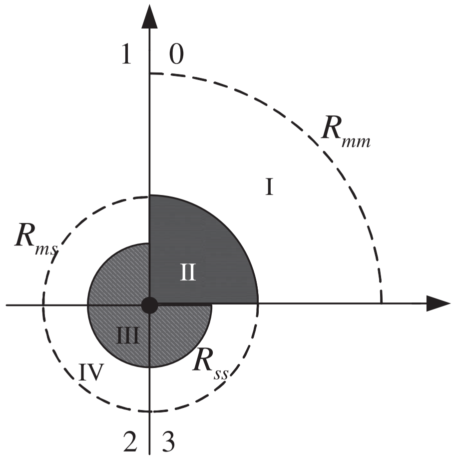

Fig. 3. This is an example when the switched antenna has four directions, which are numbered from 0 to 3 counterclockwise as shown in the figure. A receiver R_i is beamforming in direction 0 with main lobe gain G_m and in other directions with side lobe gain G_s.

where E_1, E_2, and E_3 denote the events that T_j and R_i both beamform to each other, either T_j or R_i beamforms to the other node (but not both), and neither T_j nor R_i beamforms to the other node, respectively.

Consider the single hop from a transmitter T_i to a receiver R_i. Fig. 3 shows an example that R_i is equipped with a directional antenna with four directions and it is beamforming in direction 0 where T_i is located. The big point at the center of the graph denotes the center of the disk containing the cell in which R_i is located. The distance from another point in the area to this big point is denoted by r. We also denote by R_{mm}, R_{ms}, and R_{ss} the ranges within which an interfering Voronoi cell containing another transmitter $T_j (j \neq i)$ must be located in events E_1, E_2, and E_3, respectively. Areas I, II, III, and IV denote the areas where $R_{ms} < r \leq R_{mm}$ in direction 0, where $0 < r \leq R_{ms}$ in direction 0, where $0 < r \leq R_{ss}$ in directions 1, 2, and 3, and where $R_{ss} < r \leq R_{ms}$ in directions 1, 2, and 3, respectively.

Let $a = (\frac{G_s}{G_m})^{\frac{1}{\alpha}}$. By choosing

$$\begin{aligned} R_{mm} &= 6\rho(n) + (3+\Delta)r(n), \\ R_{ms} &= 6\rho(n) + (2 + a + a\Delta)r(n), \\ R_{ss} &= 6\rho(n) + (2 + a^2 + a^2\Delta)r(n), \end{aligned}$$

we can have the following lemma:

Lemma 2. *The transmitters in a Voronoi cell not fully contained in ranges R_{mm}, R_{ms}, and R_{ss}, cannot interfere with the reception of R_i in case of E_1, E_2, and E_3, respectively. Besides, the reception of corresponding receivers of those transmitters is not interfered by T_i, either.*

Proof. Consider a transmission from another transmitter T_j to another receiver R_j. (T_j and R_j also denote the vectors from the center of the disk containing the cell where R_i is located to the two nodes, respectively.) If T_j is in a Voronoi cell not fully contained in ranges R_{mm}, then according to Lemma 1, we have

$$|T_j - R_i| \geq R_{mm} - 4\rho(n) - 2\rho(n) = (3+\Delta)r(n),$$

which conforms with the Directional Protocol Model in case of E_1. Besides, we also have

$$|T_j - T_i| \geq |T_j - R_i| - |T_i - R_i| \geq (2+\Delta)r(n),$$

and hence,

$$|T_i - R_j| \geq |T_i - T_j| - |T_j - R_j| \geq (1 + \Delta)r(n),$$

which also satisfies the Directional Protocol Model in case of E_1. Thus, we know that the transmissions from T_i to R_i and T_j to R_j do not interfere with each other in case of E_1 if T_j is in a Voronoi cell not contained in range R_{mm}.

Following a similar way, we can also prove that these two transmissions do not interfere with each other in case of E_2 and E_3, if T_j is in a Voronoi cell not contained in ranges R_{ms} and R_{ss}, respectively. □

Since each node randomly beamforms in one of the N directions, we can obtain that the nodes in Areas I and IV can interfere R_i with a probability of $\frac{1}{N}$, and those in Areas II and III can interfere R_i with a probability of 1, respectively. Thus, the maximum expected interference area in random DTDR networks, denoted by S_I^{dd}, is

$$S_I^{dd} = \frac{1}{N}\pi R_{ms}^2 + \frac{N-1}{N}\pi R_{ss}^2 + \frac{1}{N}\pi\left(R_{mm}^2 - R_{ms}^2\right)\cdot\frac{1}{N} + \frac{N-1}{N}\pi\left(R_{ms}^2 - R_{ss}^2\right)\cdot\frac{1}{N},$$

which can be simplified as

$$S_I^{dd} = O\left(\pi\rho^2(n)\left(1 + a\frac{N-1}{N}\right)^2(1+\Delta)^2\right).$$

So, the maximum number of cells in the interference area, denoted by c_2^{dd}, is

$$c_2^{dd} = \frac{S_I^{dd}}{\pi\rho^2(n)} \sim O\left(\left(1 + a\frac{N-1}{N}\right)^2\cdot(1+\Delta)^2\right).$$

Thus, by Lemma 2, we can have the following lemma:

Lemma 3. *In random DTDR networks, there is a schedule for transmitting packets such that in every c_2^{dd} slots, each cell in the tessellation V_n gets one slot in which to transmit, and such that all transmissions are successfully received within a distance $r(n)$ from their transmitters.*

We choose the routing strategy so that the routes of packets approximate the straight line connecting the source and destination. Let L_i denote the straight line connecting the source X_i and the destination Y_i. Then, we can have the following lemma [8]:

Lemma 4. *There is a $\delta'(n) \to 0$ such that*

$$\mathrm{Prob}\left(\sup_{V\in V_n}(\text{Number of lines } L_i \text{ intersecting } V) \leq c_3\sqrt{n\log n}\right) \geq 1 - \delta'(n).$$

Note that the traffic handled by a cell is proportional to the number of lines passing through it. Since each line carries traffic of rate $\lambda(n)$ bits/second, we have the following bound:

Lemma 5. *There is a $\delta'(n) \to 0$ such that*

$$\mathrm{Prob}\left(\sup_{V\in V_n}(\text{Traffic needing to be carried by cell } V) \leq c_3\lambda(n)\sqrt{n\log n}\right) \geq 1 - \delta'(n).$$

This means that the rate at which each cell needs to transmit is less than $c_3\lambda(n)\sqrt{n\log n}$ with high probability. Let W denote the channel capacity. This rate can be accommodated by all cells if it is less than the rate available, i.e., if

$$c_3\lambda(n)\sqrt{n\log n} \leq \frac{W}{c_2^{dd}}. \tag{5}$$

So, we can have the following theorem:

Theorem 1. *For random DTDR networks employing multihop relay schemes, there is a deterministic constant $0 < c < +\infty$, not depending on n, Δ, or W, such that*

$$\lambda(n) = \frac{N^2}{[N + a(N-1)]^2}\cdot\frac{cW}{(1+\Delta)^2\sqrt{n\log n}} \text{ bits/second}$$

is feasible with high probability.

3.2 Random OTDR Networks

Different from that in DTDR networks, we use another Directional Protocol Model in OTDR networks based on the Physical Model. Specifically, suppose a node T_i omnidirectionally transmits to a node R_i which receives by beamforming to T_i. Then, the positions of every other transmitters $T_j (j \neq i)$ simultaneously transmitting should satisfy:

$$\begin{cases} |T_j - R_i| \geq (1 + \triangle)r(n), & \text{if } E_1', \\ |T_j - R_i| \geq (1 + \triangle)r(n)\cdot\left(\frac{G_s}{G_m}\right)^{\frac{1}{\alpha}}, & \text{if } E_2', \end{cases}$$

where E_1' and E_2' denote the events that R_i beamforms to T_j and that R_i does not beamform to T_j, respectively. The quantity $\triangle$ has the same meaning as before.

Along the line in Section 3.1, we can obtain that the maximum interference area in random OTDR networks, denoted by S_I^{od}, is

$$S_I^{od} = O\left(\pi\rho^2(n)\left(1 + a\frac{N-1}{N}\right)(1+\Delta)^2\right),$$

and hence, the maximum number of interfered cells, denoted by c_2^{od}, is

$$c_2^{od} = \frac{S_I^{od}}{\pi\rho^2(n)} \sim O\left(\left(1 + a\frac{N-1}{N}\right)\cdot(1+\Delta)^2\right).$$

Substituting c_2^{dd} in (5) by c_2^{od}, we can have the following theorem:

Theorem 2. *For random OTDR networks employing multihop relay schemes, there is a deterministic constant $0 < c < +\infty$, not depending on n, Δ, or W, such that*

$$\lambda(n) = \frac{N}{N + a(N-1)}\cdot\frac{cW}{(1+\Delta)^2\sqrt{n\log n}} \text{ bits/second}$$

is feasible with a high probability.

3.3 Random DTOR Networks

In this case, the results are the same as in Section 3.2.

3.4 Random OTOR Networks

By setting N to 1 in Theorem 1 or Theorem 2, we can obtain the feasible throughput in OTOR networks.

Theorem 3. *For random OTOR networks employing multihop relay schemes, there is a deterministic constant $0 < c < +\infty$ not depending on n, Δ, or W, such that*

$$\lambda(n) = \frac{cW}{(1+\Delta)^2\sqrt{n\log n}} \text{ bits/second}$$

is feasible with high probability.

Note that the result in Theorem 3 is consistent with the result obtained in [8].

4 An Upper Bound on the Throughput Capacity of Random Networks—Multihop Relay

In this section, we derive an upper bound on the throughput capacity of random networks using directional antennas, when multihop relay schemes are employed.

4.1 Random DTDR Networks

Recall the Protocol Model mentioned in Section 3. Suppose that T_i and T_j transmit to R_i and R_j, respectively, at the same time. Then, we have

$$\begin{aligned}|R_i - R_j| &\geq |T_i - R_j| - |T_i - R_i| \\ &\geq (1+\Delta)|T_j - R_j| - |T_i - R_i|.\end{aligned}$$

Similarly,

$$\begin{aligned}|R_j - R_i| &\geq |T_j - R_i| - |T_j - R_j| \\ &\geq (1+\Delta)|T_i - R_i| - |T_j - R_j|.\end{aligned}$$

Adding these two inequalities together, we can get

$$|R_i - R_j| \geq \frac{\Delta}{2}(|T_i - R_i| + |T_j - R_j|) = \frac{\Delta}{2}\cdot 2r(n).$$

This means that the disks of radius $\frac{\Delta}{2}r(n)$ centered at the receivers are essentially disjoint, which we define as "*Disjoint Disks.*"

Thus, S_E, the area occupied by a single-hop's transmission, which we call "*Exclusion Area,*" is

$$S_E = \pi\left(\frac{\Delta}{2}r(n)\right)^2 = \frac{\pi\Delta^2}{4}r^2(n).$$

Note that the Protocol Model can only be used in OTOR networks and that in OTOR networks, Exclusion Area is equal to the area of disjoint disks. However, we contend that *this is not necessarily true when directional antennas are used*. Instead, in directional networks, Exclusion Area could be much smaller than the area of disjoint disks.

Consider the single hop from T_i to R_i. We use Fig. 3 as an example where a receiver R_i denoted by the big point at the center beamforms in direction 0. Let R_{mm}, R_{ms}, and R_{ss} denote the ranges within which this receiver R_i would be interfered by another transmitter T_j's ($j \neq i$) transmission in events E_1, E_2, and E_3, respectively. Referring to the Directional Protocol Model for random DTDR networks introduced in Section 3.1, we have $R_{mm} = (1+\Delta)r(n)$, $R_{ms} = (1+\Delta)r(n)\cdot(\frac{G_s}{G_m})^{\frac{1}{\alpha}}$, and $R_{ss} = (1+\Delta)r(n)\cdot(\frac{G_s}{G_m})^{\frac{2}{\alpha}}$. Note that the nodes in Areas I and IV can interfere R_i with a probability of $\frac{1}{N}$, and those in Areas II and III can interfere R_i with a probability of 1, respectively. Since the transmitters located in Areas I-IV can possibly interfere with the reception of R_i, we define these areas as "*Interference Area.*" Intuitively, *Exclusion Area is contained in Interference Area*. So, we choose the intersection part of Disjoint Disk and Interference Area[1] as Exclusion Area in DTDR networks, which we denote by S_E^{dd} and calculate as follows:

Let $a = (\frac{G_s}{G_m})^{\frac{1}{\alpha}}$.

1. When $a > \frac{1}{\sqrt{2}}$, i.e., $\frac{\Delta}{2}r(n) < R_{ss}$, we have

$$S_E^{dd} = \pi\left(\frac{\Delta}{2}r(n)\right)^2 = \frac{\pi\Delta^2}{4}r^2(n).$$

2. When $\frac{1}{2} < a \leq \frac{1}{\sqrt{2}}$, we have
   a. If $\Delta > \frac{2a^2}{1-2a^2}$, i.e., $R_{ss} \leq \frac{\Delta}{2}r(n) < R_{ms}$, then

$$\begin{aligned}S_E^{dd} = &\frac{1}{N}\pi\left(\frac{\Delta}{2}r(n)\right)^2 + \frac{N-1}{N}\pi R_{ss}^2 \\ &+ \frac{N-1}{N}\pi\left[\left(\frac{\Delta}{2}r(n)\right)^2 - R_{ss}^2\right]\frac{1}{N}.\end{aligned}$$

   b. If $0 < \Delta < \frac{2a^2}{1-2a^2}$, i.e., $\frac{\Delta}{2}r(n) < R_{ss}$, then we get the same result as that in 1.
3. When $0 < a < \frac{1}{2}$, we have
   a. If $\Delta > \frac{2a}{1-2a}$, i.e., $R_{ms} \leq \frac{\Delta}{2}r(n) < R_{mm}$, then

$$\begin{aligned}S_E^{dd} = &\frac{1}{N}\pi R_{ms}^2 + \frac{1}{N}\pi\left[\left(\frac{\Delta}{2}r(n)\right)^2 - R_{ms}^2\right]\cdot\frac{1}{N} \\ &+ \frac{N-1}{N}\pi R_{ss}^2 + \frac{N-1}{N}\pi\left(R_{ms}^2 - R_{ss}^2\right)\cdot\frac{1}{N}.\end{aligned}$$

   b. If $0 < \Delta < \frac{2a}{1-2a}$, i.e., $R_{ss} \leq \frac{\Delta}{2}r(n) < R_{ms}$, then we arrive at same result as that in 2a.

Thus, we have the following lemma:

Lemma 6. *In random DTDR networks, the number of simultaneous transmissions on the channel is no more than $N_{max} = 1/S_E^{dd}$ under the Directional Protocol Model.*

Note that each transmission is of W bits/second. By adding all the transmissions taking place at the same time, we can find that the network can at most accommodate $N_{max}\cdot W$ bits per second.

Let $\overline{L}$ denote the mean length of a line connecting two independently and uniformly distributed points on the disk plane. Then, the mean length of the path of packets is at least $\overline{L} - o(1)$ since there is always a node within a distance $o(1)$ of a point with high probability. Thus, the mean number of hops taken by a packet is at least $\frac{\overline{L}-o(1)}{r(n)}$. Since each source generates $\lambda(n)$ bits/second, there are n sources, and the total number of bits per second served by the entire network needs to be at least $\frac{(\overline{L}-o(1))n\lambda(n)}{r(n)}$. To ensure that all the required traffic is carried, we therefore need

1. If nodes in an area S interfere with the receiver R_i with a probability p, then the effective Interference Area is $p\cdot S$.

$$\frac{(\overline{L} - o(1))n\lambda(n)}{r(n)} \leq N_{max}W.$$

Recall that $r(n) > \sqrt{\frac{\log n}{\pi n}}$ is necessary for OTOR networks to guarantee connectivity with high probability [7]. Since we assume a transmitter and its intended receiver can beamform to each other to carry out the transmission when needed, then $r(n) > \sqrt{\frac{\log n}{\pi n}}$ is also necessary for DTDR networks to guarantee connectivity. Thus, after simple calculations, we have

$$\lambda(n) \leq \begin{cases} \frac{c'W}{\Delta^2\sqrt{n\log n}} \\ \quad \text{when } a > \frac{1}{\sqrt{2}}, \\ \quad \text{or when } \frac{1}{2} < a \leq \frac{1}{\sqrt{2}}, \quad 0 < \Delta < \frac{2a^2}{1-2a^2}, \\ \frac{1}{4\left[a^4\left(\frac{N-1}{N}\right)^2(1+\Delta)^2+\frac{2N-1}{N^2}\frac{\Delta^2}{4}\right]}\frac{c'W}{\sqrt{n\log n}} \\ \quad \text{when } \frac{1}{2} < a \leq \frac{1}{\sqrt{2}}, \quad \Delta > \frac{2a^2}{1-2a^2}, \\ \quad \text{or when } 0 < a < \frac{1}{2}, \quad 0 < \Delta < \frac{2a}{1-2a}, \\ \frac{1}{4\left[\left(a^2\frac{N-1}{N}+\frac{1}{N}\right)^2(1+\Delta)^2-\frac{(2+3\Delta)(2+\Delta)}{4N^2}\right]}\frac{c'W}{\sqrt{n\log n}} \\ \quad \text{when } 0 < a < \frac{1}{2}, \quad \Delta > \frac{2a}{1-2a}, \end{cases} \tag{6}$$

where $0 < c' < +\infty$, not depending on n, Δ, or W.

Moreover, since there are at most $\frac{n}{2}$ transmissions that can be carried out at the same time, we have $N_{max} \leq \frac{n}{2}$. Thus, we can obtain

$$\frac{(\overline{L} - o(1))n\lambda(n)}{r(n)} \leq \frac{n}{2}W,$$

i.e.,

$$\lambda(n) \leq c'W\sqrt{\frac{\log n}{n}}. \tag{7}$$

We finally arrive at the following theorem:

Theorem 4.

1. *For random DTDR networks employing multihop relay schemes, an upper bound on the per-node throughput capacity is* $\lambda(n) = \min\{\lambda_1(n), \lambda_2(n)\}$ *bits/second, where* $\lambda_1(n)$ *and* $\lambda_2(n)$ *are shown in (6) and (7), respectively.*
2. *An upper bound on the per-node throughput capacity is*

$$\min\left\{\frac{c'N^2W}{\Delta^2\sqrt{n\log n}}, c'W\sqrt{\log n/n}\right\} \text{ bits/second}$$

when $a = o(1/\sqrt{N})$, *and*

$$\min\left\{\frac{c'a^{-4}W}{4(1+\Delta)^2\sqrt{n\log n}}, c'W\sqrt{\log n/n}\right\} \text{ bits/second}$$

when $a = \Omega(1/\sqrt{N})$, *respectively, where* $0 < c' < +\infty$, *not depending on* n, Δ, *or* W.

4.2 Random OTDR Networks

Let $a = (\frac{G_s}{G_m})^{\frac{1}{\alpha}}$. Similarly, we can obtain that

$$\lambda(n) = \begin{cases} \frac{c'W}{\Delta^2\sqrt{n\log n}} \\ \quad \text{when } a > \frac{1}{2}, \\ \quad \text{or when } 0 < a \leq \frac{1}{2}, \quad 0 < \Delta \leq \frac{2a}{1-2a}, \\ \frac{1}{4\left[\frac{N-1}{N}a^2(1+\Delta)^2+\frac{\Delta^2}{4N}\right]}\frac{c'W}{\sqrt{n\log n}} \\ \quad \text{when } 0 < a \leq \frac{1}{2}, \quad \Delta > \frac{2a}{1-2a}, \end{cases} \tag{8}$$

and

$$\lambda(n) \leq c'W\sqrt{\frac{\log n}{n}}. \tag{9}$$

Thus, we can have the theorem below:

Theorem 5.

1. *For random OTDR networks employing multihop relay schemes, an upper bound on the per-node throughput capacity is* $\lambda(n) = \min\{\lambda_1(n), \lambda_2(n)\}$ *bits/second, where* $\lambda_1(n)$ *and* $\lambda_2(n)$ *are shown in (8) and (9), respectively.*
2. *An upper bound on the per-node throughput capacity is*

$$\min\left\{\frac{c'NW}{\Delta^2\sqrt{n\log n}}, c'W\sqrt{\log n/n}\right\} \text{ bits/second}$$

when $a = o(1/\sqrt{N})$, *and*

$$\min\left\{\frac{c'a^{-2}W}{4(1+\Delta)^2\sqrt{n\log n}}, c'W\sqrt{\log n/n}\right\} \text{ bits/second}$$

when $a = \Omega(1/\sqrt{N})$, *respectively, where* $0 < c' < +\infty$, *not depending on* n, Δ, *or* W.

4.3 Random DTOR Networks

For random DTOR networks, the results are the same as those for random OTDR networks shown in Section 4.2.

4.4 Random OTOR Networks

By setting N to 1 in Theorem 4 or Theorem 5, we can have the following result:

Theorem 6. *For random OTOR networks employing multihop relay schemes, an upper bound on the per-node throughput capacity is*

$$\lambda(n) = \frac{c'W}{\Delta^2\sqrt{n\log n}}$$

bits/second, where $0 < c' < +\infty$, *not depending on* n, Δ, *or* W.

Note that the result in Theorem 6 is also consistent with the result obtained in [8].

4.5 More Discussions

From Theorems 1-2 and Theorems 4-5, we observe that if directional antennas have nonnegligible side lobe gain, i.e., when $a = \Theta(1)$, the per-node throughput capacity in random DTDR, in OTDR, and in DTOR networks is bounded by $\Theta(1/\sqrt{n\log n})$. But, if directional antennas have very small side lobe gain, i.e., when $a = o(1)$, the per-node throughput capacity of DTDR networks, of OTDR networks, and of DTOR networks, is all upper bounded by $\sqrt{\log n/n}$, a gain factor of $\log n$ compared to that of OTOR networks shown in Theorem 6.

Thus, combining with the results in Section 3, we arrive at the following corollary:

Corollary 1. *The throughput capacity of random DTDR, OTDR, and DTOR networks cannot scale as the number of nodes when using multihop relay. Furthermore, the per-node throughput capacity when using directional antennas is upper bounded by* $O(\sqrt{\log n/n})$ *when* $(G_s/G_m)^{1/\alpha} = o(1)$, *i.e., when $G_s/G_m = o(1)$, and is bounded by $\Theta(1/\sqrt{n \log n})$ when $G_s/G_m = \Theta(1)$.*

Moreover, we notice that *although using directional antennas in random networks with multihop relay schemes cannot make the networks scale, it can help save a lot of energy in the networks*. Recall the power propagation model introduced in Section 2.2. Transmission range $r(n)$ is defined as follows:

$$P_t C \frac{G_t G_r}{(r(n))^\alpha} = RX_{th},$$

where RX_{th} is the receiver sensitivity. Let P_{DD}, P_{DO}, P_{OD}, and P_{OO} denote the transmission power in DTDR, DTOR, OTDR, and OTOR networks, respectively. Then, we can obtain that

$$P_{DD} = \frac{1}{G_m^2} P_{OO},$$
$$P_{DO} = P_{OD} = \frac{1}{G_m} P_{OO}.$$

Recall the directional antenna model in Section 2.1. According to (1), we can have

$$G_m \cdot x + G_s \cdot (1 - x) = \eta, \tag{10}$$

where $x = \frac{1}{2}(1 - \cos\frac{\pi}{N})$. As a result, we obtain that $G_m \le \frac{\eta}{x}$, and hence,

$$P_{DD} = \Omega\Big(\sin^4 \frac{\pi}{2N}\Big) P_{OO}, \tag{11}$$

$$P_{DO} = P_{OD} = \Omega\Big(\sin^2 \frac{\pi}{2N}\Big) P_{OO}. \tag{12}$$

In addition, as N goes large, we have

$$P_{DD} = \Omega\Big(\frac{1}{N^4}\Big) P_{OO},$$
$$P_{DO} = P_{OD} = \Omega\Big(\frac{1}{N^2}\Big) P_{OO}.$$

The lower bounds become tighter as the side lobe gain G_s gets smaller.

5 A Lower Bound on the Throughput Capacity of Random Networks—One-Hop Delivery

We have shown in Sections 3 and 4 that using directional antennas cannot make the throughput capacity of the network scale as the number of nodes if we employ multihop relay schemes. In this section and next section, we investigate whether enabling only single-hop transmissions directly from a transmitter to a receiver can achieve this goal. Here, we begin by presenting a lower bound on the throughput capacity when using one-hop relay schemes. As in Section 3, we also assume a transmitter and a receiver can beamform to each other to carry out the transmission.

5.1 Random DTDR Networks

To ensure direct transmissions between an arbitrary transmitter-receiver pair, we choose the transmission range when a transmitter and a receiver both beamform to each other, denoted by r_{mm}, to be

$$r_{mm} = r(n) = 2/\sqrt{\pi}.$$

Thus, the transmission range when either a transmitter or a receiver (but not both) beamforms to the other, and when neither of them beamforms to the other, denoted by r_{ms} and r_{ss}, respectively, can be obtained by

$$r_{ms} = (2/\sqrt{\pi}) \cdot a = 2a/\sqrt{\pi},$$
$$r_{ss} = (2/\sqrt{\pi}) \cdot a^2 = 2a^2/\sqrt{\pi},$$

where $a = (\frac{G_s}{G_m})^{\frac{1}{\alpha}}$.

Consider a single hop from T_i to R_i. Fig. 3 shows an example when a receiver denoted by the big point at the center beamforms in direction 0. Denote by R_{mm}, R_{ms}, and R_{ss} the ranges within which this receiver R_i would be interfered by another transmitter T_j's $(j \ne i)$ transmission in events E_1, E_2, and E_3, respectively. According to the Directional Protocol Model, we have

$$R_{mm} = (1 + \Delta) \cdot (2/\sqrt{\pi}),$$
$$R_{ms} = (1 + \Delta) \cdot (2a/\sqrt{\pi}),$$
$$R_{ss} = (1 + \Delta) \cdot (2a^2/\sqrt{\pi}).$$

Note that the nodes in Areas I and IV can interfere R_i with a probability of $\frac{1}{N}$, and those in Areas II and III can interfere R_i with a probability of 1, respectively. Thus, the maximum interference area in random DTDR networks within which the transmitters in other transmissions will interfere with R_i, denoted by S_{IR}^{dd}, can be calculated by

$$\begin{aligned} S_{IR}^{dd} &= \frac{1}{N}\pi R_{ms}^2 + \frac{N-1}{N}\pi R_{ss}^2 \\ &\quad + \frac{1}{N}\pi\big(R_{mm}^2 - R_{ms}^2\big) \cdot \frac{1}{N} + \frac{N-1}{N}\pi\big(R_{ms}^2 - R_{ss}^2\big) \cdot \frac{1}{N} \\ &= 4\left[\frac{1}{N}(1+\Delta) + \frac{N-1}{N}(1+\Delta)a^2\right]^2. \end{aligned}$$

Similarly, we can have that the maximum interference area in random DTDR networks within which the receivers in other transmissions will be interfered by T_i, denoted by S_{IT}^{dd}, is equal to S_{IR}^{dd}. Thus, the interference area of the transmission from T_i to R_i, denoted by S_I^{dd}, is

$$S_I^{dd} \le S_{IR}^{dd} + S_{IT}^{dd} = 8\left[\frac{1}{N}(1+\Delta) + \frac{N-1}{N}(1+\Delta)a^2\right]^2.$$

So, the maximum number of nodes in the interference area, denoted by m^{dd}, is

$$m^{dd} = n \cdot S_I^{dd} = 8n\left[\frac{1}{N}(1+\Delta) + \frac{N-1}{N}(1+\Delta)a^2\right]^2.$$

Recall that in Section 2.3, we choose random source-destination pairs so that each node is a source node for one flow and a destination node for at most $O(1)$ flows. So, the maximum number of transmissions engaged by the nodes in the interference area, denoted by t^{dd}, is $t^{dd} \le (1 + c_0)m^{dd}$, where c_0 is a deterministic constant, i.e., $c_0 = O(1)$.

Besides, note that $t^{dd} \ge 2$ since there are at least two transmissions engaged by the nodes in the interference area, i.e., the transmission from T_i to R_i and the transmission from R_i to T_i. So, we have

$$t^{dd} = \max\left\{8(1+c_0)n\left[\frac{1}{N}(1+\Delta)+\frac{N-1}{N}(1+\Delta)a^2\right]^2, 2\right\}.$$

Lemma 7. *In random DTDR networks, there is a schedule for transmitting packets such that in every t^{dd} slot, each node in the network gets one slot in which to transmit, and such that all transmissions are successfully received within a distance $r(n)$ from their transmitters.*

Thus, we can obtain that

$$\lambda(n) \leq \frac{W}{t^{dd}} = \min\left\{\frac{c''W}{n\left[\frac{1}{N}(1+\Delta)+\frac{N-1}{N}(1+\Delta)a^2\right]^2}, \frac{W}{2}\right\}.$$

This leads to the following theorem:

Theorem 7.

1. *For random DTDR networks employing one-hop delivery schemes, the per-node throughput*

$$\lambda(n) = \min\left\{\frac{c''W}{n\left[\frac{1}{N}(1+\Delta)+\frac{N-1}{N}(1+\Delta)a^2\right]^2}, \frac{W}{2}\right\}$$

bits/second is achievable, where $0 < c'' < +\infty$ is a deterministic constant not depending on n, Δ, or W.

2. *An achievable per-node throughput is*

$$\min\left\{\frac{c''N^2W}{(1+\Delta)^2 n}, \frac{W}{2}\right\} \text{ bits/second}$$

when $a = o(1/\sqrt{N})$, and

$$\min\left\{\frac{c''a^{-4}W}{(1+\Delta)^2 n}, \frac{W}{2}\right\} \text{ bits/second}$$

when $a = \Omega(1/\sqrt{N})$, respectively.

5.2 Random OTDR and DTOR Networks

Similarly, we can have the following theorem. The detailed proof is omitted due to space limit.

Theorem 8.

1. *For random OTDR and DTOR networks employing one-hop delivery schemes, the per-node throughput*

$$\lambda(n) = \min\left\{\frac{c''W}{n(1+\Delta)^2\left(\frac{1}{N}+\frac{N-1}{N}a^2\right)}, \frac{W}{2}\right\}$$

bits/second is achievable, where $0 < c'' < +\infty$ is a deterministic constant not depending on n, Δ, or W.

2. *An achieveable per-node throughput is*

$$\min\left\{\frac{c''NW}{(1+\Delta)^2 n}, \frac{W}{2}\right\} \text{ bits/second}$$

when $a = o(1/\sqrt{N})$, and

$$\min\left\{\frac{c''a^{-2}W}{(1+\Delta)^2 n}, \frac{W}{2}\right\} \text{ bits/second}$$

when $a = \Omega(1/\sqrt{N})$, respectively.

5.3 Random OTOR Networks

By setting N to 1 in Theorem 7 or Theorem 8, we can have the following theorem:

Theorem 9. *For random OTOR networks employing one-hop delivery schemes, the per-node throughput*

$$\lambda(n) = \frac{c''W}{n(1+\Delta)^2} \text{ bits/second}$$

is feasible, where $0 < c'' < +\infty$ is a deterministic constant not depending on n, Δ, or W.

6 An Upper Bound on the Throughput Capacity of Random Directional Networks—One-Hop Delivery

6.1 Random DTDR Networks

In random DTDR networks, the Exclusion Area denoted by S_E^{dd} can be calculated in the same way as that in Section 4.1. Substituting $r(n)$ by $2/\sqrt{\pi}$, we obtain that

$$S_E^{dd} = \begin{cases} \Delta^2 \\ \quad \text{when } a > \frac{1}{\sqrt{2}}, \\ \quad \text{or when } \frac{1}{2} < a \leq \frac{1}{\sqrt{2}}, \quad 0 < \Delta < \frac{2a^2}{1-2a^2}, \\ 4\left[a^4\left(\frac{N-1}{N}\right)^2(1+\Delta)^2 + \frac{2N-1}{N^2}\frac{\Delta^2}{4}\right] \\ \quad \text{when } \frac{1}{2} < a \leq \frac{1}{\sqrt{2}}, \quad \Delta > \frac{2a^2}{1-2a^2}, \\ \quad \text{or when } 0 < a < \frac{1}{2}, \quad 0 < \Delta < \frac{2a}{1-2a}, \\ 4\left[\left(\frac{N-1}{N}a^2 + \frac{1}{N}\right)^2(1+\Delta)^2 - \frac{(2+3\Delta)(2+\Delta)}{4N^2}\right] \\ \quad \text{when } 0 < a < \frac{1}{2}, \quad \Delta > \frac{2a}{1-2a}. \end{cases}$$

Since each node does not need to relay packets for other nodes, to ensure all the required traffic can be carried by the networks, we need

$$n\lambda(n) \leq \frac{W}{S_E^{dd}},$$

i.e.,

$$\lambda(n) \leq \begin{cases} \frac{c'W}{n\Delta^2} \\ \quad \text{when } a > \frac{1}{\sqrt{2}}, \\ \quad \text{or when } \frac{1}{2} < a \leq \frac{1}{\sqrt{2}}, \quad 0 < \Delta < \frac{2a^2}{1-2a^2}, \\ \frac{c'W}{n\left[a^4\left(\frac{N-1}{N}\right)^2(1+\Delta)^2+\frac{2N-1}{N^2}\frac{\Delta^2}{4}\right]} \\ \quad \text{when } \frac{1}{2} < a \leq \frac{1}{\sqrt{2}}, \quad \Delta > \frac{2a^2}{1-2a^2}, \\ \quad \text{or when } 0 < a < \frac{1}{2}, \quad 0 < \Delta < \frac{2a}{1-2a}, \\ \frac{c'W}{n\left[\left(\frac{N-1}{N}a^2+\frac{1}{N}\right)^2(1+\Delta)^2-\frac{(2+3\Delta)(2+\Delta)}{4N^2}\right]} \\ \quad \text{when } 0 < a < \frac{1}{2}, \quad \Delta > \frac{2a}{1-2a}. \end{cases} \quad (13)$$

Besides, we also have $1/S_E^{dd} \leq \frac{n}{2}$, i.e., $S_E^{dd} \geq \frac{2}{n}$, because there are at most $\frac{n}{2}$ concurrent transmissions. So, we have that

$$\lambda(n) \leq \frac{W}{2}. \quad (14)$$

Finally, we arrive at the following theorem:

Theorem 10.

1. *For random DTDR networks employing one-hop delivery schemes, an upper bound on the per-node throughput capacity is $\lambda(n) = \min\{\lambda_1(n), \lambda_2(n)\}$*

bits/second, where $\lambda_1(n)$ and $\lambda_2(n)$ are shown in (13) and (14), respectively.

2. *An achieveable per-node throughput is*

$$\min\left\{\frac{c'N^2W}{\Delta^2 n}, \frac{W}{2}\right\} \text{ bits/second}$$

when $a = o(1/\sqrt{N})$, and

$$\min\left\{\frac{c'a^{-4}W}{\Delta^2 n}, \frac{W}{2}\right\} \text{ bits/second}$$

when $a = \Omega(1/\sqrt{N})$, respectively, where $0 < c' < +\infty$, not depending on n, Δ, or W.

6.2 Random OTDR and DTOR Networks

Following the steps in Section 6.1, we can have

$$\lambda(n) \leq \begin{cases} \frac{c'W}{n\Delta^2} \\ \quad \text{when } a > \frac{1}{\sqrt{2}}, \\ \quad \text{or when } 0 < a \leq \frac{1}{\sqrt{2}}, \quad 0 < \Delta < \frac{2a}{1-2a}, \\ \frac{c'W}{n\left[\frac{\Delta^2}{4N} + \frac{N-1}{N}a^2(1+\Delta)^2\right]} \\ \quad \text{when } 0 < a \leq \frac{1}{2}, \quad \Delta > \frac{2a}{1-2a}, \end{cases} \tag{15}$$

and

$$\lambda(n) \leq \frac{W}{2}. \tag{16}$$

So, we can obtain the following theorem:

Theorem 11.

1. *For random OTDR and DTOR networks employing one-hop delivery schemes, an upper bound on the per-node throughput capacity is $\lambda(n) = \min\{\lambda_1(n), \lambda_2(n)\}$ bits/second, where $\lambda_1(n)$ and $\lambda_2(n)$ are shown in (15) and (16), respectively.*
2. *An upper bound on the per-node throughput capacity is*

$$\min\left\{\frac{c'NW}{\Delta^2 n}, \frac{W}{2}\right\} \text{ bits/second}$$

when $a = o(1/\sqrt{N})$ and

$$\min\left\{\frac{c'a^{-2}W}{\Delta^2 n}, \frac{W}{2}\right\} \text{ bits/second}$$

when $a = \Omega(1/\sqrt{N})$, respectively, where $0 < c' < +\infty$, not depending on n, Δ, or W.

6.3 Random OTOR Networks

By setting N to 1 in Theorem 10 or Theorem 11, we have the following theorem:

Theorem 12. *For random OTOR networks employing one-hop delivery schemes, an upper bound on the per-node throughput capacity is*

$$\lambda(n) = \frac{c'W}{n\Delta^2} \text{ bits/second,}$$

where $0 < c' < +\infty$, not depending on n, Δ, or W.

6.4 More Discussions

Combining with the results derived in Section 5, we find that with one-hop delivery schemes, using directional antennas can make the per-node throughput capacity scale as $\Theta(\frac{N^2W}{n})$ and $\Theta(\frac{NW}{n})$ in random DTDR networks, respectively, when $a = o(1/\sqrt{N})$ and as $\Theta\left(\frac{a^{-4}W}{n}\right)$ and $\Theta\left(\frac{a^{-2}W}{n}\right)$ in random DTDR networks and random DTOR, OTDR networks, respectively, when $a = \Omega(1/\sqrt{N})$. However, if the side lobe gain cannot be neglected, the per-node throughput capacity of random DTDR, DTOR, and OTDR networks can only scale as $\Theta(\frac{W}{n})$, and the use of directional antennas can only improve the throughput capacity by a constant factor compared to that of random OTOR networks. Thus, we have the following corollary:

Corollary 2. *Let $a = N^x$, where $x \leq 0$. With one-hop delivery schemes, the throughput capacity of random DTDR, OTDR, and DTOR networks can scale as the number of nodes n if the beam number N scales as $\sqrt{n}$, n, and n, respectively, when $x < -\frac{1}{2}$, or as $n^{-\frac{1}{4x}}$, $n^{-\frac{1}{2x}}$, and $n^{-\frac{1}{2x}}$, respectively, when $-\frac{1}{2} \leq x < 0$.*

Besides, similar to (11) and (12) in Section 4.5, we show in the following that using directional antennas can save a lot of energy compared to using omnidirectional antennas. Moreover, we will also show that *no matter in random dense networks or in random extended networks where n nodes are randomly distributed in a disk of area n, the power consumption of directional antennas with one-hop delivery can be upper bounded by a constant.*

Consider one-hop direct transmissions in dense networks. The average transmission range would be $\Theta(1)$. Thus, according to the power propagation model, we have

$$P_t C \frac{G_t G_r}{1^\alpha} = RX_{th}.$$

Let P^d_{DD}, P^d_{DO}, and P^d_{OD} denote the transmission power in dense DTDR, DTOR, and OTDR networks, respectively. Then,

$$P^d_{DD} = \Theta\left(\frac{1}{G_m^2}\right), P^d_{DO} = P^d_{OD} = \Theta\left(\frac{1}{G_m}\right).$$

According to (10), we have

$$P^d_{DD} = \Omega\left(\sin^4\frac{\pi}{2N}\right) = O(1),$$
$$P^d_{DO} = P^d_{OD} = \Omega\left(\sin^2\frac{\pi}{2N}\right) = O(1).$$

The lower bounds become tighter as G_s gets smaller.

Next, consider the same problem in extended networks. The average transmission range would be $\Theta(\sqrt{n})$. Let P^e_{DD}, P^e_{DO}, and P^e_{OD} denote the transmission power in extended DTDR, DTOR, and OTDR networks, respectively. Then, we can obtain that

$$P^e_{DD} = \Theta\left(\frac{n^{\frac{\alpha}{2}}}{G_m^2}\right),$$
$$P^e_{DO} = P^e_{OD} = \Theta\left(\frac{n^{\frac{\alpha}{2}}}{G_m}\right).$$

Especially when side lobe directional antenna gain is negligible, we have

$$G_m \approx \frac{\eta}{\sin^2\frac{\pi}{2N}} \geq \frac{4\eta N^2}{\pi^2}. \tag{17}$$

Thus,

$$P_{DD}^{e} = O\left(\frac{n^{\frac{\alpha}{2}}}{N^4}\right), P_{DO}^{e} = P_{OD}^{e} = O\left(\frac{n^{\frac{\alpha}{2}}}{N^2}\right),$$

which means

$$\begin{aligned} P_{DD}^{e} &= O(1), && \text{when } N = \Omega\left(n^{\frac{\alpha}{8}}\right), \\ P_{DO}^{e} = P_{OD}^{e} &= O(1), && \text{when } N = \Omega\left(n^{\frac{\alpha}{4}}\right). \end{aligned}$$

7 Trade-Offs between Transmission Range and Throughput in Random Directional Networks

In random OTOR networks, it has been shown that the maximum throughput is achieved when the transmission range is chosen to be the smallest one which can ensure the network connectivity, i.e., $\Theta(\sqrt{\log n/n})$. On the contrary, as we have shown in Sections 3-6, in DTDR, OTDR, and DTOR networks, using larger transmission range can give us higher throughput. In this section, we investigate the trade-offs between transmission range and throughput in random directional networks.

7.1 Random DTDR Networks

Recall the results in Section 4.1, we have

$$\frac{(\overline{L} - o(1))n\lambda(n)}{r(n)} \leq \frac{W}{S_E^{dd}},$$

where

$$S_E^{dd} = \begin{cases} \frac{\pi\Delta^2}{4} r^2(n) \\ \quad \text{when } a > \frac{1}{\sqrt{2}}, \\ \quad \text{or when } \frac{1}{2} < a \leq \frac{1}{\sqrt{2}}, \quad 0 < \Delta < \frac{2a^2}{1-2a^2}, \\ \pi\left[a^4\left(\frac{N-1}{N}\right)^2(1+\Delta)^2 + \frac{2N-1}{N^2}\frac{\Delta^2}{4}\right] r^2(n) \\ \quad \text{when } \frac{1}{2} < a \leq \frac{1}{\sqrt{2}}, \quad \Delta > \frac{2a^2}{1-2a^2}, \\ \quad \text{or when } 0 < a < \frac{1}{2}, \quad 0 < \Delta < \frac{2a}{1-2a}, \\ \pi\left[\left(\frac{N-1}{N}a^2 + \frac{1}{N}\right)^2(1+\Delta)^2 - \frac{(2+3\Delta)(2+\Delta)}{4N^2}\right] r^2(n) \\ \quad \text{when } 0 < a < \frac{1}{2}, \quad \Delta > \frac{2a}{1-2a}. \end{cases}$$

Thus, we can obtain that

$$\lambda(n) \leq \begin{cases} \frac{c'W}{nr(n)\Delta^2} \\ \quad \text{when } a > \frac{1}{\sqrt{2}}, \\ \quad \text{or when } \frac{1}{2} < a \leq \frac{1}{\sqrt{2}}, 0 < \Delta < \frac{2a^2}{1-2a^2}, \\ \frac{c'W}{nr(n)\left[a^4\left(\frac{N-1}{N}\right)^2(1+\Delta)^2 + \frac{2N-1}{N^2}\frac{\Delta^2}{4}\right]} \\ \quad \text{when } \frac{1}{2} < a \leq \frac{1}{\sqrt{2}}, \Delta > \frac{2a^2}{1-2a^2}, \\ \quad \text{or when } 0 < a < \frac{1}{2}, 0 < \Delta < \frac{2a}{1-2a}, \\ \frac{c'W}{nr(n)\left[\left(\frac{N-1}{N}a^2 + \frac{1}{N}\right)^2(1+\Delta)^2 - \frac{(2+3\Delta)(2+\Delta)}{4N^2}\right]} \\ \quad \text{when } 0 < a < \frac{1}{2}, \Delta > \frac{2a}{1-2a}. \end{cases} \tag{18}$$

Besides, since there are at most $\frac{n}{2}$ concurrent transmissions, we also have that

$$\frac{(\overline{L} - o(1))n\lambda(n)}{r(n)} \leq \frac{n}{2}W,$$

i.e.,

$$\lambda(n) \leq \frac{r(n)W}{2}. \tag{19}$$

Finally, we arrive at the following theorem:

Theorem 13.

1. *For random DTDR networks, an upper bound on the per-node throughput capacity is* $\lambda(n) = \min\{\lambda_1(n), \lambda_2(n)\}$ *bits/second, where* $\lambda_1(n)$ *and* $\lambda_2(n)$ *are shown in (18) and (19), respectively.*
2. *An upper bound on the per-node throughput capacity is*

$$\min\left\{\frac{c'N^2W}{\Delta^2 nr(n)}, \frac{r(n)W}{2}\right\} \text{ bits/second}$$

when $a = o(1/\sqrt{N})$, *and*

$$\min\left\{\frac{c'a^{-4}W}{(1+\Delta)^2 nr(n)}, \frac{r(n)W}{2}\right\} \text{ bits/second}$$

when $a = \Omega(1/\sqrt{N})$, *respectively, where* $0 < c' < +\infty$, *not depending on* n, Δ, *or* W.

7.2 Random OTDR and DTOR Networks

Similarly, we can obtain that

$$\lambda(n) \leq \begin{cases} \frac{c'W}{nr(n)\Delta^2} \\ \quad \text{when } a > \frac{1}{\sqrt{2}}, \\ \quad \text{or when } 0 < a \leq \frac{1}{\sqrt{2}}, \quad 0 < \Delta < \frac{2a}{1-2a}, \\ \frac{c'W}{nr(n)\left[\frac{\Delta^2}{4N} + \frac{N-1}{N}a^2(1+\Delta)^2\right]} \\ \quad \text{when } 0 < a \leq \frac{1}{2}, \quad \Delta > \frac{2a}{1-2a}, \end{cases} \tag{20}$$

and

$$\lambda(n) \leq \frac{r(n)W}{2}. \tag{21}$$

So, we can obtain the following theorem:

Theorem 14.

1. *For random OTDR and DTOR networks employing one-hop delivery schemes, an upper bound on the per-node throughput capacity is* $\lambda(n) = \min\{\lambda_1(n), \lambda_2(n)\}$ *bits/second, where* $\lambda_1(n)$ *and* $\lambda_2(n)$ *are shown in (20) and (21), respectively.*
2. *An upper bound on the per-node throughput capacity is*

$$\min\left\{\frac{c'NW}{\Delta^2 nr(n)}, \frac{r(n)W}{2}\right\} \text{ bits/second}$$

when $a = o(1/\sqrt{N})$, *and*

$$\min\left\{\frac{c'a^{-2}W}{(1+\Delta)^2 nr(n)}, \frac{r(n)W}{2}\right\} \text{ bits/second}$$

when $a = \Omega(1/\sqrt{N})$, *respectively, where* $0 < c' < +\infty$, *not depending on* n, Δ, *or* W.

7.3 Random OTOR Networks

By setting N to 1 in Theorem 13 or Theorem 14, we have the following theorem:

Theorem 15. *For random OTOR networks employing one-hop delivery schemes, an upper bound on the per-node throughput capacity is*

$$\lambda(n) = \frac{c'W}{nr(n)\Delta^2} \text{ bits/second},$$

where $0 < c' < +\infty$, *not depending on* n, Δ, *or* W.

7.4 More Discussions

According to the results derived above, we have the following corollary:

Corollary 3. *In random directional networks, we can have higher throughput if we use larger transmission range and keep the side lobe antenna gain low at the same time.*

Besides, according to the power propagation model, we have

$$P_t C \frac{G_t G_r}{(r(n))^{\alpha}} = RX_{th}.$$

Let P_{DD}, P_{DO}, and P_{OD} denote the transmission power in DTDR, DTOR, and OTDR networks, respectively. Then,

$$P_{DD} = \Theta\left(\frac{r^{\alpha}(n)}{G_m^2}\right),$$
$$P_{DO} = P_{OD} = \Theta\left(\frac{r^{\alpha}(n)}{G_m}\right).$$

When side lobe directional antenna gain is negligible, according to (17), we have

$$P_{DD} = O\left(\frac{r^{\alpha}(n)}{N^4}\right), P_{DO} = P_{OD} = O\left(\frac{r^{\alpha}(n)}{N^2}\right),$$

which leads to

$$\begin{aligned} &P_{DD} = O(1), && \text{when } N = \Omega\left(r^{\frac{\alpha}{4}}(n)\right),\\ &P_{DO} = P_{OD} = O(1), && \text{when } N = \Omega\left(r^{\frac{\alpha}{2}}(n)\right). \end{aligned}$$

8 A Lower Bound on the Transport Capacity of Arbitrary Networks

In this section, by presenting an achievable throughput in a certain scenario, we derive a lower bound on the transport capacity of arbitrary networks using directional antennas. We assume there are n nodes on a planar disk of unit area which use the same transmission power. As we will show later, the transmitter-receiver pairs are fixed after placing the nodes. So, we can make the transmitters and their corresponding receivers beamform to each other to carry out the transmission.

8.1 Arbitrary DTDR Networks

Theorem 16. *There is a placement of nodes and an assignment of traffic patterns such that, under the Protocol Model, arbitrary DTDR networks can achieve*

$$\frac{nW}{4(1+\Delta)\left(\sqrt{\frac{n}{2N\lfloor\frac{N}{3}\rfloor+4N}}+\sqrt{2\pi}\right)} \text{ bit-meters per second},$$

when we neglect the side lobe gain.

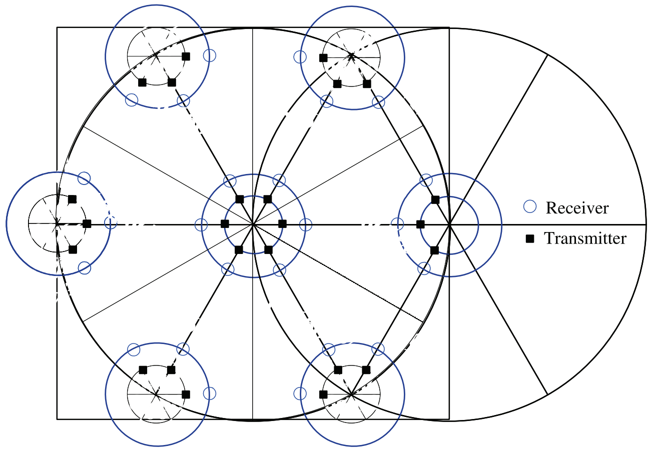

Fig. 4. An example for illustrating the locations of the nodes in arbitrary DTDR networks when $N = 6$.

Proof. Recall that the domain is a disk of unit area, i.e., of radius $\frac{1}{\sqrt{\pi}}$ in the plane. Let $L = 2(1+\Delta)r$, $r_1 = \frac{\Delta}{4}r$, $r_2 = \frac{3\Delta}{4}r$, and $r_3 = r + r_1$. With the center of disk located at the origin, place transmitters at locations

$$\left[pL + r_1\cos\left(\frac{\pi}{2} + \frac{(2i-1)\pi}{N}\right),\right.$$
$$\left. qL + r_1\sin\left(\frac{\pi}{2} + \frac{(2i-1)\pi}{N}\right)\right],$$

and

$$\left[pL + \frac{L}{2}\cos\frac{(3N+4i\pm 4j-2)\pi}{2N} + r_1\cos\frac{(N+4i-2)\pi}{2N},\right.$$
$$\left. qL + \frac{L}{2}\sin\frac{(3N+4i\pm 4j-2)\pi}{2N} + r_1\sin\frac{(N+4i-2)\pi}{2N}\right],$$

and place receivers at locations

$$\left[pL + r_3\cos\left(\frac{\pi}{2} + \frac{(2i-1)\pi}{N}\right),\right.$$
$$\left. qL + r_3\sin\left(\frac{\pi}{2} + \frac{(2i-1)\pi}{N}\right)\right],$$

and

$$\left[pL + \frac{L}{2}\cos\frac{(3N+4i\pm 4j-2)\pi}{2N} + r_3\cos\frac{(N+4i-2)\pi}{2N},\right.$$
$$\left. qL + \frac{L}{2}\sin\frac{(3N+4i\pm 4j-2)\pi}{2N} + r_3\sin\frac{(N+4i-2)\pi}{2N}\right],$$

where p, q, i, and j are integers, and $i \in [0, N-1]$, $j \in [0, \frac{N}{6}]$. One example with $N = 6$ is shown in Fig. 4. Transmitters use power control to make $r_{mm} = r$, where r_{mm} is the transmission range when a transmitter and a receiver beamform to each other. Thus, each transmitter can transmit to the receiver which is exactly at a distance of r away, without interference from any other transmitter-receiver pairs.

By doing this, the nodes are arranged in squares with length L, where $NUM = N \times (2 + \lfloor\frac{N}{3}\rfloor)$ pairs of transmitter-receiver are located. All such squares that

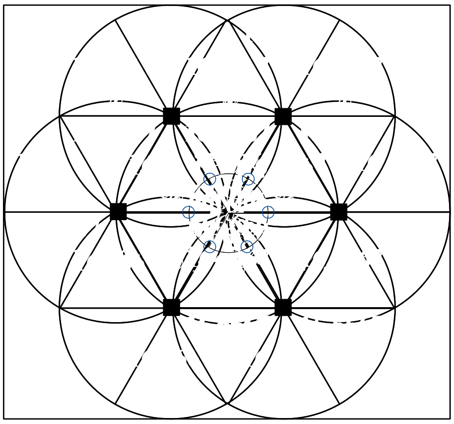

Fig. 5. An example for illustrating the locations of the nodes in arbitrary OTDR networks when $N = 6$.

intersect with a disk of radius $\frac{1}{\sqrt{\pi}} - \sqrt{2}L$ are entirely contained in the domain disk of radius $\frac{1}{\sqrt{\pi}}$. So, we have

$$\frac{\pi\left(\frac{1}{\sqrt{\pi}} - \sqrt{2}L\right)^2}{L^2} \cdot NUM = \frac{n}{2},$$

which gives $r = \frac{1}{2(1+\Delta)(\sqrt{\frac{n}{2 \cdot NUM}} + \sqrt{2\pi})}$. Thus, the transport capacity for this configuration is

$$\frac{n}{2}Wr = \frac{nW}{4(1+\Delta)\left(\sqrt{\frac{n}{2N\lfloor\frac{N}{3}\rfloor + 4N}} + \sqrt{2\pi}\right)}$$

bit-meters per second, which completes the proof. □

8.2 Arbitrary OTDR Networks

Theorem 17. *There is a placement of nodes and an assignment of traffic patterns such that, under the Protocol Model, OTDR networks can achieve*

$$\frac{nW}{8(1+\frac{\Delta}{2})\left(\sqrt{\frac{n}{2N}} + \sqrt{2\pi}\right)} \text{bit-meters per second,}$$

when we neglect the side lobe gain.

Proof. Let $L = 4(r + r_1)$, and $r_1 = \frac{\Delta}{2}r$. With the center of disk located at the origin, place transmitters at locations

$$\left[pL + (r + r_1)\cos\left(\frac{\pi}{2} + \frac{(2i-1)\pi}{N}\right), \right.$$
$$\left. qL + (r + r_1)\sin\left(\frac{\pi}{2} + \frac{(2i-1)\pi}{N}\right)\right],$$

and place receivers at locations

$$\left[pL + r_1\cos\left(\frac{\pi}{2} + \frac{(2i-1)\pi}{N}\right), \right.$$
$$\left. qL + r_1\sin\left(\frac{\pi}{2} + \frac{(2i-1)\pi}{N}\right)\right],$$

where p, q, and i are integers, and $i \in [0, N-1]$. One example with $N = 6$ is shown in Fig. 5. Transmitters use power control to make $r_m = r$, where r_m is the transmission range when a receiver beamforms to a transmitter. Each transmitter can transmit to the nearest receiver which is exactly at a distance of r away, without interference from any other transmitter-receiver pairs.

Placed in this way, the nodes are arranged in squares with length L, where N pairs of transmitter-receiver are located. Following the process in Section 8.1, we can get

$$r = \frac{1}{\left(\sqrt{\frac{n}{2N}} + \sqrt{2\pi}\right)(4 + 2\Delta)}.$$

So, the transport capacity is

$$\frac{n}{2}Wr = \frac{nW}{8(1+\frac{\Delta}{2})\left(\sqrt{\frac{n}{2N}} + \sqrt{2\pi}\right)}.$$

□

8.3 Arbitrary DTOR Networks

In this case, exchanging the positions of transmitters and receivers, we can obtain the same result as that in OTDR networks.

8.4 Arbitrary OTOR Networks

Theorem 18. *There is a placement of nodes and an assignment of traffic patterns such that, under the Protocol Model, OTOR networks can achieve ([8])*

$$\frac{W}{(1+2\Delta)} \frac{2\lfloor\frac{n}{4}\rfloor}{\left(\sqrt{\lfloor\frac{n}{4}\rfloor} + \sqrt{2\pi}\right)} \text{bit-meters per second.}$$

8.5 More Discussions

We show in this section that without side lobe gain, the lower bounds on transport capacity per node for arbitrary DTDR, OTDR, DTOR, and OTOR networks are $\Omega(\frac{N}{\sqrt{n}}W)$, $\Omega(\sqrt{\frac{N}{n}}W)$, $\Omega(\sqrt{\frac{N}{n}}W)$, and $\Omega(\frac{W}{\sqrt{n}})$, respectively. Thus, we have the following corollary:

Corollary 4. *Without side lobe gain, arbitrary DTDR, OTDR, and DTOR networks can scale when the beam number N of directional antennas increases as fast as $\sqrt{n}$, n, and n, respectively.*

9 Conclusion

In this paper, we study the throughput capacity in random networks and the transport capacity in arbitrary networks when directional antennas are used. In random directional networks, on the one hand, we find that if multihop relay schemes are used in the network, the per-node throughput capacity cannot be improved much by using directional antennas compared to that using omnidirectional antennas. The capacity gain is at most $\log n$ compared to the omnidirectional case. On the other hand, we also show that if one-hop delivery schemes are used, the per-node throughput capacity in DTDR, OTDR, and DTOR networks can scale if directional antennas have very small side lobe gain so that $G_s/G_m = o(1)$, and the beam number N increases as the number of nodes goes large in the network. Moreover, we also investigate the trade-offs between transmission range and throughput when directional antennas are used, and find that we can have higher throughput if we use larger transmission range. In addition, we also show that using directional antennas can help save a lot of energy, and that even when one-hop delivery schemes are used, the transmission power for each node can be upper bounded by a constant.

In arbitrary directional networks, we present a lower bound on the transport capacity by deriving an achievable throughput in a certain scenario. We find that arbitrary directional networks can scale if without side lobe antenna gain.

Finally, we notice that in order for both random and arbitrary networks to scale, the number of beams of directional antennas needs to increase as the number of nodes increases. We hope this can be realized as the directional antenna technology progresses although the current technology can only accommodate a limited number of beams.

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