MATHEMATICAL RELATIONSHIPS

Table B1-1. Gradient Operation on a Scalar Function, C

Coordinates	**Components of ∇C**
Rectangular	x: $\frac{\partial C}{\partial x}$, y: $\frac{\partial C}{\partial y}$, z: $\frac{\partial C}{\partial z}$
Cylindrical	r: $\frac{\partial C}{\partial r}$, θ: $\frac{1}{r}\frac{\partial C}{\partial \theta}$, z: $\frac{\partial C}{\partial z}$
Spherical	r: $\frac{\partial C}{\partial r}$, θ: $\frac{1}{r}\frac{\partial C}{\partial \theta}$, ϕ: $\frac{1}{r\sin\theta}\frac{\partial C}{\partial \phi}$

Table B1-2. Divergence Operation on a Vector Field, u

Coordinates	$\nabla \cdot \mathbf{u}$
Rectangular	$\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}+\frac{\partial u_z}{\partial z}$
Cylindrical	$\frac{1}{r}\frac{\partial}{\partial r}\left(r u_r\right)+\frac{1}{r}\frac{\partial u_\theta}{\partial \theta}+\frac{\partial u_z}{\partial z}$
Spherical	$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 u_r\right)+\frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(u_\theta \sin\theta\right)+\frac{1}{r\sin\theta}\frac{\partial u_\phi}{\partial \phi}$

Table B1-3. Laplacian Operation on a Scalar Function, C

Coordinates	$\nabla^2 C$
Rectangular	$\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}+\frac{\partial^2 C}{\partial z^2}$
Cylindrical	$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 C}{\partial \theta^2}+\frac{\partial^2 C}{\partial z^2}$
Spherical	$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial C}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial C}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 C}{\partial \phi^2}$

Table C4-1. Laplace Transforms of Common Operations

Operation	Time Domain	Laplace Domain
1st Derivative	$f'(t)$	$s\tilde{f}(s)-f(0)$
nth Derivative	$f^{(n)}(t)$	$s^n\tilde{f}(s)-s^{n-1}f(0)-s^{n-2}f'(0)-\ldots-sf^{n-2}(0)-f^{(n-1)}(0)$
Integration	$\int_0^t f(\tau)d\tau$	$\frac{\tilde{f}(s)}{s}$
Initial Value Theorem	$\lim_{t\to 0} f(t)$	$\lim_{s\to\infty} s\tilde{f}(s)$
Final Value Theorem	$\lim_{t\to\infty} f(t)$	$\lim_{s\to 0} s\tilde{f}(s)$
Shift of t	$f(t-t_o)$	$e^{-st_o}\tilde{f}(s)$
Shift of s	$e^{at}f(t)$	$\tilde{f}(s-a)$
Convolution Integral	$\int_0^t f(t-\tau)g(\tau)d\tau$	$\tilde{f}(s)*\tilde{g}(s)$

Table C4-2 Laplace Transforms of Common Functions

Time Domain, f(t)	Laplace Domain, $\tilde{f}(s)$
a	$\frac{a}{s}$
t, t^n (n=1,2,…)	$\frac{1}{s^2}$, $\frac{n!}{s^{n+1}}$
e^{at}, $t^n e^{at}$ $(n=1,2,...)$	$\frac{1}{s-a}$, $\frac{n!}{(s-a)^{n+1}}$
$\frac{e^{at}-e^{bt}}{a-b}$, $\frac{ae^{at}-be^{bt}}{b-a}$	$\frac{1}{(s-a)(s-b)}$, $\frac{s}{(s-a)(s-b)}$
sinh(at), cosh(at)	$\frac{a}{s^2-a^2}$, $\frac{s}{s^2-a^2}$
sin(ωt+ϕ), cos(ωt+ϕ)	$\frac{s\sin(\phi)+\omega\cos(\phi)}{s^2+\omega^2}$, $\frac{s\cos(\phi)-\omega\sin(\phi)}{s^2+\omega^2}$
$\operatorname{erfc}\left(\frac{a}{2\sqrt{t}}\right)$	$\frac{e^{-a\sqrt{s}}}{s}$
Unit Impulse: $\delta(t-t_o)$	e^{-st_o}
Unit Step: $H(t-t_o)$	$\frac{e^{-st_o}}{s}$

* a and b are real distinct constants.