MASS TRANSPORT EQUATIONS

Table 13.1-1. Continuity Equation for an Incompressible Fluid

Rectangular coordinates: $$\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}+\frac{\partial u_z}{\partial z}=0$$
Cylindrical coordinates: $$\frac{1}{r}\frac{\partial}{\partial r}\left(r u_r\right)+\frac{1}{r}\frac{\partial u_\theta}{\partial \theta}+\frac{\partial u_z}{\partial z}=0$$
Spherical coordinates: $$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 u_r\right)+\frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(u_\theta \sin\theta\right)+\frac{1}{r\sin\theta}\frac{\partial u_\phi}{\partial \phi}=0$$

Table 9.1-1 Components of the Molar Flux Vector: Constant Mass Density

Rectangular coordinates: $N_{i,x} = u_z C_i - \mathcal{D}_i \frac{\partial C_i}{\partial x}, \quad N_{i,y} = u_y C_i - \mathcal{D}_i \frac{\partial C_i}{\partial y}, \quad N_{i,z} = u_z C_i - \mathcal{D}_i \frac{\partial C_i}{\partial z}$
Cylindrical coordinates: $N_{i,r} = u_r C_i - \mathcal{D}_i \frac{\partial C_i}{\partial r}, \quad N_{i,\theta} = u_\theta C_i - \mathcal{D}_i \frac{1}{r}\frac{\partial C_i}{\partial \theta}, \quad N_{i,z} = u_z C_i - \mathcal{D}_i \frac{\partial C_i}{\partial z}$
Spherical coordinates: $N_{i,r} = u_r C_i - \mathcal{D}_i \frac{\partial C_i}{dr}, \quad N_{i,\theta} = u_\theta C_i - \mathcal{D}_i \frac{1}{r}\frac{\partial C_i}{d\theta}, \quad N_{i,\varphi} = u_\varphi C_i - \mathcal{D}_i \frac{1}{r\sin\theta}\frac{\partial C_i}{\partial \varphi}$

Table 15.2-1 Species Concentration Equations: Constant ρ and $\mathcal{D}_A$

Rectangular coordinates: $\frac{\partial C_A}{\partial t} + \left(u_x \frac{\partial C_A}{\partial x} + u_y \frac{\partial C_A}{\partial y} + u_z \frac{\partial C_A}{\partial z} \right) = \mathcal{D}_A \left(\frac{\partial^2 C_A}{\partial x^2} + \frac{\partial^2 C_A}{\partial y^2} + \frac{\partial^2 C_A}{\partial z^2} \right) + R_A$
Cylindrical coordinates: $\frac{\partial C_A}{\partial t} + \left(u_r \frac{\partial C_A}{\partial r} + u_\theta \frac{1}{r}\frac{\partial C_A}{\partial \theta} + u_z \frac{\partial C_A}{\partial z} \right) = \mathcal{D}_A \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial C_A}{\partial r} \right) + \frac{1}{r^2}\frac{\partial^2 C_A}{\partial \theta^2} + \frac{\partial^2 C_A}{\partial z^2} \right] + R_A$
Spherical coordinates: $\frac{\partial C_A}{\partial t} + \left(u_r \frac{\partial C_A}{\partial r} + u_\theta \frac{1}{r}\frac{\partial C_A}{\partial \theta} + u_\phi \frac{1}{r\sin\theta}\frac{\partial C_A}{\partial \phi} \right) = \mathcal{D}_A \left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial C_A}{\partial r} \right) + \frac{1}{r^2 \sin\theta}\left(\sin\theta \frac{\partial^2 C_A}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 C_A}{\partial \phi^2} \right] + R_A$

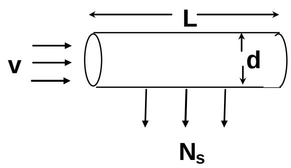
Table 9.1-2 Unidirectional Radial Transport

Constant Mass Density, ρ	
Axisymmetric ($S_r \propto r$)	$$\frac{\partial\left(r u_r\right)}{\partial r}=0$$ $$\frac{\partial C_i}{\partial t}+u\frac{\partial C_i}{\partial r}=\frac{1}{r}\frac{\partial}{\partial r}\left(\mathcal{D}_i r\frac{\partial C_i}{\partial r}\right)+R_i$$
Spherically Symmetric ($S_r \propto r^2$)	$$\frac{\partial\left(r^2 u_r\right)}{\partial r}=0$$ $$\frac{\partial C_i}{\partial t}+u\frac{\partial C_i}{\partial r}=\frac{1}{r^2}\frac{\partial}{\partial r}\left(\mathcal{D}_i r^2\frac{\partial C_i}{\partial r}\right)+R_i$$
Constant Molar Density c	
Axisymmetric ($S_r \propto r$)	$$\frac{\partial\left(r u_r^*\right)}{\partial r}=\frac{r}{c}\sum_{k=1}^{I}R_k$$ $$\frac{\partial C_i}{\partial t}+u^*\frac{\partial C_i}{\partial r}=\frac{1}{r}\frac{\partial}{\partial r}\left(\mathcal{D}_i r\frac{\partial C_i}{\partial r}\right)+R_i-\frac{C_i}{c}\sum_{k=1}^{I}R_k$$
Spherically Symmetric ($S_r \propto r^2$)	$$\frac{\partial\left(r^2 u_r^*\right)}{\partial r}=\frac{r^2}{c}\sum_{k=1}^{I}R_k$$ $$\frac{\partial C_i}{\partial t}+u^*\frac{\partial C_i}{\partial r}=\frac{1}{r^2}\frac{\partial}{\partial r}\left(\mathcal{D}_i r^2\frac{\partial C_i}{\partial r}\right)+R_i-\frac{C_i}{c}\sum_{k=1}^{I}R_k$$

Table 12.1-1 Dimensionless, Surface-Averaged, Mass Transfer Coefficients

$(Re_d = \rho u d/\mu : \ Re_L = \rho u L/\mu : Sh_d = k_s d/\mathcal{D}_s : Sh_L = k_s L/\mathcal{D}_s : Sc = \mu/\rho\mathcal{D}_s)$

1. FULLY-DEVELOPED FLOW THROUGH CIRCULAR TUBES (Middleman, 1998)

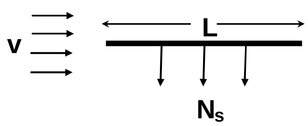


a) Laminar Flow With Developing Concentration Profile
$Sh_d = 1.62\ [Re_d Sc(d/L)]^{1/3}$: $Re_d < 2100$, $L/d < 0.025\ Re_d Sc$

b) Laminar Flow With Fully-Developed Concentration Profile
$Sh_d = 3.66$: $Re_d < 2100$, $L/d > 0.025\ Re_d Sc$

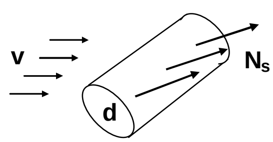
c) Turbulent Flow
$Sh_d = 0.023\ Re_d^{0.83} Sc^{0.44}$: $Re_d > 4000$

2. FLOW PARALLEL TO A FLAT PLATE (Treybal, 1980)



$Sh_L = 0.664\ Re_L^{1/2}\ Sc^{1/3}$

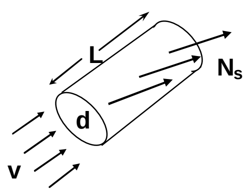
3. FLOW PERPENDICULAR TO CYLINDERS



a) Single Cylinder (Treybal, 1980)
$Sh_d = (0.35 + 0.34\ Re_d^{0.5} + 0.15\ Re_d^{0.58})\ Sc^{0.3}$

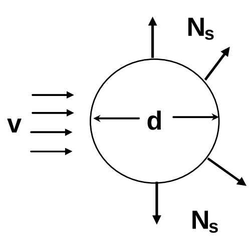
b) Evenly Spaced Array of Cylinders (Cussler, 1997, p 226-227)*
$Sh_d = 0.80 Re_d^{0.47} Sc^{1/3}$

4. FLOW PARALLEL TO EVENLY-SPACED ARRAYS OF CYLINDERS (Yang and Cussler, 1986)*



a) $Sh_d = 1.25\ [Re_d\ (d/L)]^{0.93}\ Sc^{0.33}$: $\varepsilon = 0.97$
b) $Sh_d = 0.022\ Re_d^{0.60} Sc^{0.33}$: $\varepsilon = 0.74$
c) $Sh_d = 0.24$: $\varepsilon = 0.60$

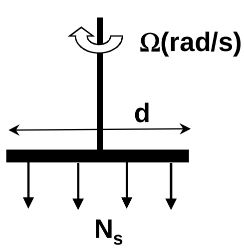
5. FLOW PAST SPHERES



a) Single Sphere (Cussler, 1997, p 226-227)
$Sh_d = 2.0 + 0.6\ Re_d^{1/2}\ Sc^{1/3}$

b) Spheres Randomly Packed in a Tube (Treybal, 1980)*
$Sh_d = (0.25/\varepsilon)\ Re_d^{0.69}\ Sc^{1/3}$

6. MASS TRANSFER FROM A SPINNING CIRCULAR DISK (Cussler, 1997, p 226-227)



$Sh_d = 1.24\ Re_\Omega^{1/2}\ Sc^{1/3}$
$(Re_\Omega \equiv \rho d^2 \Omega / 4\mu)$

*u=superficial velocity through packed tube or across tube bank
ε=void fraction between particle packing or tube banks

Table 18.3-1 Moments of a Unit Impulse-Response With Zero Loss in SISO Models

Model	Transfer Function $\tilde{g}=\tilde{C}_{out}/\tilde{C}_{in}$	Mean Appearance Time $\bar{t}_g$	Variance σ_g^2
1) Transport Delay	Eq. 18.2-17	$\frac{V}{Q}$	0
2) Single Compartment (Fig. 13.1-5)	Eq. 18.1-26	$\frac{V}{Q}$	$\left(\frac{V}{Q}\right)^2$
3) J Series Compartments of Unequal Volumes V_j (Fig. 13.2-1; J=2)	Eq. 18.2-11	$\frac{V}{Q}=\frac{\sum_{j=1}^{J} V_j}{Q}$	$\sum_{j=1}^{J}\left(\frac{V_j}{Q}\right)^2$
4) J Series Compartments Of Equal Volumes V_J (Fig. 13.2-3)	Eq. 18.2-12	$\frac{V}{Q}=\frac{JV_J}{Q}$	$J\left(\frac{V_J}{Q}\right)^2$
5) 2 Parallel Compartments of Volumes V_1 and V_2 Without Interaction (Fig. 13.2-5)	Eq. 18.2-22	$\frac{V}{Q}=\frac{V_1+V_2}{Q_1+Q_2}$	$\left(\frac{V}{Q}\right)^2 \times\left[\frac{2Q}{Q_1}\left(\frac{V_1}{V}\right)^2+\frac{2Q}{Q_2}\left(\frac{V_2}{V}\right)^2-1\right]$
6) 2 Parallel Compartments of Volumes V_1 and V_2 With Flow Interaction (Fig. 13.2-7)	Eq. 18.2-35	$\frac{V}{Q}=\frac{V_1+V_2}{Q}$	$\left(\frac{V}{Q}\right)^2\left[1+2\frac{Q}{Q_{12}}\left(\frac{V_2}{V}\right)^2\right]$
7) 2 Parallel Compartments of Volumes V_1 and V_2 With Diffusion Interaction (Fig. 13.2-9)	Eq. 18.2-46	$\frac{V_1+V_2/\lambda^{12}}{Q}$	$\left(\frac{V_1+V_2/\lambda^{12}}{Q}\right)^2 \times\left[1+\frac{2Q}{P_{12}S}\left(\frac{V_2}{\lambda^{12}V_1+V_2}\right)^2\right]$