PART I

CHAPTER 1.

Problem 1-1 Estimation of Cell Radius

The total number of cells in a human body is estimated to be 37 trillion. Assuming spherical cells, determine the average radius of a human cell based on an intracellular volume of 25L.

Problem 1-2 Estimation of Alveolus Radius

From section 1.2 in the textbook, the lung has a volume at end-inhalation that is 5L/2=2.5L and a surface area of 50-100m^2. The organ contains about 3×10^6 alveoli. Given that the large majority of the gas-filled space in the lungs occurs within its alveoli, estimate the surface-to-volume ratio of a single alveolus. If an alveolus is modeled as a sphere, then estimate its equivalent radius.

Problem 1-3 Respiratory Quotient and Calorific Equivalent

Stearic acid $CH_3(CH_2)_{16}CO_2H$ is a saturated fatty acid with a heat of reaction for complete combustion of ΔH_r=-9.60 kcal/g.

(a) Convert the heat of reaction from cal/g to J/mol.
(b) Write the stoichiometric reaction equation for the complete combustion of stearic acid.
(c) Find the respiratory quotient (RQ) of this compound.
(d) Determine its calorific equivalent (CE) .Note that the volume of each mole of an ideal gas is 25.4 L at body temperature and pressure.
(e) How do the CE and RQ of stearic acid compare to triolein (Eq. 1.1-2), a fatty acid which has more than three times as many carbon atoms in each molecule?

Problem 1-4: Whole Body Metabolism

The volumetric oxygen consumption and carbon dioxide excretion from the respiratory system of an adult person are simultaneously measured as 235 and 200 cm^3(STP)/min, respectively. In this problem, you are to assume that O_2 consumption and CO_2 production are due only to the metabolism of carbohydrates (modeled as glucose) and fats (modeled as triolean),

(a) Derive the following relations: the molar oxygen consumption rate $\dot{M}_{O_2}$ (mol/min) in terms of the nutrient consumption rates, $\dot{M}_{O_2,glucose}$ and $\dot{M}_{O_2,triolean}$; the molar carbon dioxide production rate $\dot{M}_{CO_2}$ (mol/min) in terms of $\dot{M}_{O_2,glucose}$ and $\dot{M}_{O_2,triolean}$ using their respiratory quotients, $RQ_{glucose}$ and $RQ_{triolean}$.

(b) Evaluate $\dot{M}_{O_2,glucose}$ and $\dot{M}_{O_2,triolean}$ from the two independent equations of part (a) given that $RQ_{glucose}$=1 and $RQ_{triolean}$=0.713. Note that 22.4 (liters-STP/mol) converts CE values from kJ/(L-STP) to kJ/mol.

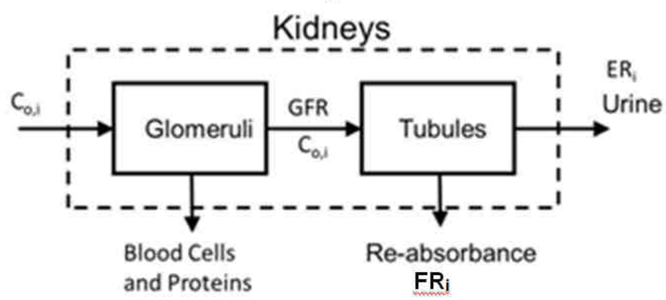
(c) Estimate the heat generated $\dot{H}$ (kJ/min) from the body by nutrient combustion using the calorific equivalents: $CE_{glucose}=21$; $CE_{triolean}=18.5$ kJ/(L-STP).

(d) What are the molar consumption rates of carbohydrates and fats, $\dot{M}_{glucose}$ and $\dot{M}_{triolean}$ (mol / min)? What are their daily mass consumption rates, $\dot{m}_{glucose}$ and $\dot{m}_{triolean}$ (g/day)?

Problem 1-5: Renal Clearance

Clearance rate of a substance i (CR_i), a performance parameter for evaluating kidney function, is defined as the volumetric rate of entering plasma from which the substance would have to be totally removed to account for its excretion rate in urine (ER_i). If $C_{o,i}$ is the entering concentration of the substance in plasma, then $ER_i = CR_i x C_{o,i}$.

Important operating parameters of a kidney are the glomerular filtration rate (GFR) of fluid from the glomeruli to the renal tubules, and the fractional reabsorbance of a substance i (FR_i) from the tubules to the renal capillaries.



(a) Using the following definition of FR_i, obtain an equation for CR_i in terms of GFR and FR_i. Assume that molecules of substance i are so small that they completely permeate the glomeruli to reach the tubules.

$$FR_i = \frac{(\text{Molar rate of i entering tubules}) - (\text{Molar rate of i exreted in urine})}{(\text{Molar rate of i entering tubules})}$$

(b) Suppose that GFR=150 liters/day and the percent of substance reabsorbed from the filtrate is 100% for glucose, 90% for potassium ion and 50% for urea. Compute the clearances for these three substances.

(c) Recompute the clearance rates from part (a) if the GFR is increased by 5% and the mass rates of reabsorption do not change.

(d) Recompute the clearance rate for potassium from part (a) if its reabsorption rate is decreases by 5% and the GFR does not change.

(e) Based on these results, compare the effect of GFR to the effect of $FR_{potassium}$ on potassium clearance.

CHAPTER 2.

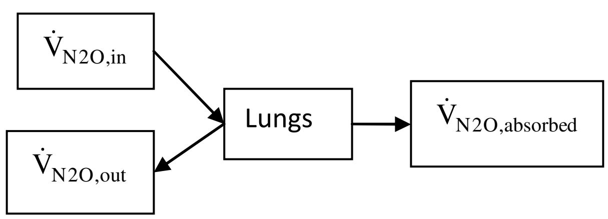
Problem 2-1: Body Composition by Underwater Weighing

A person weighing 65.3 kg on dry land is submerged in a tank of 85^oF water with mass density 0.996 kg/L and then exhales to residual volume (RV). The difference between the dry land weight and submerged weight is the weight of the displaced water (Archimedes Principle).

(a) Estimate the total (tissue plus gas) volume of the person after exhalation to RV.
(b) If the residual volume of gas in the lungs is 1.78 L (as estimated by a multibreath nitrogen washout), what is the tissue volume? What is the approximate tissue weight of the person?
(c) Relate the total tissue weight to the fat and lean tissue volumes and densities.
(d) Assume the density of fat tissue is $\rho_{fat} = 0.91$ kg/L and the density of lean tissue is ρ_{lean} = 1.10 kg/liter. Determine the volume of fat and the volume of lean tissue.
(e) What is the weight percentage of fat tissue in the body?

Problem 2-2: Pulmonary Gas Balance

While receiving nitrous oxide (N_2O) as an anesthetic, a person is breathing at a rate BR= 15 breaths per minute. On each breath, the inhaled tidal volume is V_{in}=500 ml at 27^oC and a total pressure P=101.3 kPa. The exhaled tidal volume is V_{out}=600 ml at 37^oC and P=101.3 kPa. Samples of the respired gas that are dried before being analyzed are found to contain mole fractions $y_{N2O,dry}$ of 0.10 and 0.07 in inhaled and exhaled gas, respectively.

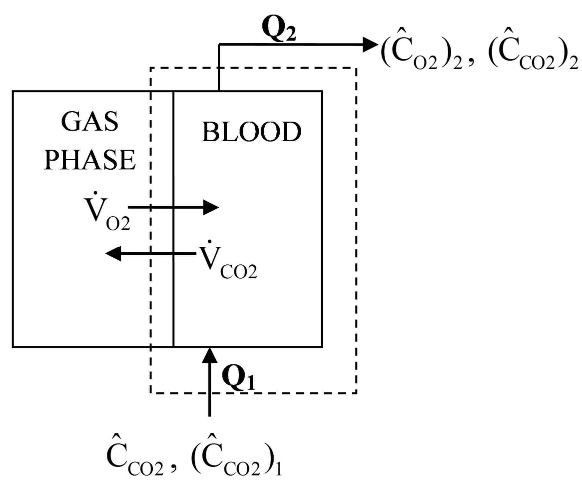


(a) Assuming an ideal gas mixture, show that the mole fraction of N_2O before drying is given by $y_{N_2O} = (P - p_w) y_{N2O,dry} / P$ where p_w is the partial pressure of water vapor in respired air before drying.
(b) Compute the mole fractions of N_2O in inhaled and in exhaled air, $y_{N2O,in}$ and $y_{N2O,out}$, given that the partial pressures of water vapor are $p_{w,in}$=3.56 kPa in inhaled air and $p_{w,out}$=6.27 kPa in exhaled air.
(c) Write an equation for the N_2O volumetric input per breath $\dot{V}_{N2O,in}$ [ml(STP) / min] in terms of $y_{N2O,in}$, BR, and V_{in}. Write a similar expression of N_2O volumetric output per breath $\dot{V}_{N2O,out}$.
(d) Evaluate $\dot{V}_{N2O,in}$ and $\dot{V}_{N2O,out}$ in units of [ml(STP)/min].

(e) Based on a steady-state N_2O volumetric rate balance around the lungs, compute the rate of N_2O absorbed $\dot{V}_{N2O,absorbed}$ [ml(STP)/min].

Problem 2-3: Extracorporeal Gas Exchange

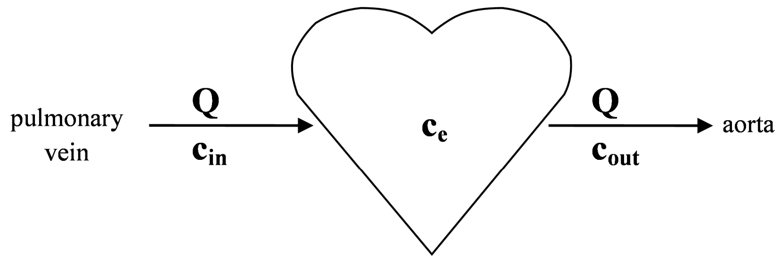
During open heart surgery, an extracoporeal heart-lung machine pumps a patient's blood through a oxygenator between the vena cava and the aorta. Within the oxygenator, O_2 is bubbled through the flowing blood where it is absorbed at a rate of $\dot{V}_{O2}$=5 mmol/min. The O_2 and CO_2 contents ($\hat{C}_{O2}, \hat{C}_{CO2}$) of blood entering the device are 15 and 53 ml(STP)/100 ml, respectively. The O_2 content in the blood exiting the device is 20 ml(STP)/100 ml. The RQ of the device is 0.90. Consider the blood in the oxygenator (excluding gas bubbles) as the control volume with volumetric input and output flows Q_1 and Q_2.



(a) Assuming that the mass density of blood is essentially constant, what is the relationship between Q_1 and Q_2 during steady state operation?
(b) Write a steady-state molar balance for O_2 around the control volume. From this, evaluate the volumetric blood flow.
(c) Write a steady-state molar balance for CO_2 around the control volume. What is the uptake relationship of $\dot{V}_{CO2}$ to $\dot{V}_{O2}$? With these equations, evaluate $(\hat{C}_{CO2})_2$.

Problem 2-4: Heart Volume Estimate by Dye Dilution

A catheter inserted into the pulmonary artery provides a continuous infusion of Evan's blue dye into the inferior vena blood that leads to a constant inlet concentration c_{in} to the right heart. The dye concentration c_{out} is subsequently monitored in the aorta. Under these conditions, Evan's blue dye is confined to the bloodstream.



(a) Perform a spatially-lumped balance to obtain a differential equation for dynamic changes of dye concentration $c_{out}(t)$. Note that preceding the continuous infusion, no dye is present.
(b) Obtain an equation for aorta dye concentration as a function of time.
(c) If it takes 2 seconds for the dye concentration in the aorta to reach 50% of its input level when the cardiac output is 6L/min, estimate the volume of blood in the heart.

Problem 2-5 Drug Release From a Cylindrical Source

Investigators have performed *in vitro* experiments of drug release from a cylindrical polymer rod of constant density ρ implanted in tissue. The rod contains drug bound to polymer such that its mass density in the rod is ρ_D is constant. As the polymer slowly degrades, the rod shrinks in radius and drug is released into surrounding tissue at the rod surface. To model drug release from a rod of initial mass M(0), we will assume that the rate of change of rod mass, M(t), is proportional to the surface area of the rod, S(t).

$$\frac{dM}{dt} = -\kappa S$$

(a) What is the relationship of M(t) to the mass density ρ, radius R(t), and length L of the rod? Assuming the length L>>R, how does this simplify the expression of S in terms of R and L?
(b) Modify the above model equation with this assumption and obtain a differential equation for the radius of the device as a function of time. The initial rod radius is R_o.
(c) Solve for R(t) and use the result to obtain M(t).
(d) Derive the relation between the amount of tracer (or drug) released, $M_D(t)$, the mass loss of the device and ρ_D. Show that the amount of tracer released has the form $M_D=\alpha t-\beta t^2$. Relate α and β to other model parameters.

Problem 2-6: Intestinal Transport and Reaction

A polysaccharide P_n is convectively transported with chyme through the small intestine where it is hydrolyzed to produce glucose according to the reaction:

$$P_n + (n-1)H_2O \rightarrow nG.$$

Glucose is produced from the polysaccharide at a molar rate per unit volume $\dot{R}_G/V$ and is absorbed across through the mucosal wall of the intestine at a molar rate per unit surface $\dot{M}_G/S$. The polysaccharide is not absorbed in the intestine.

Assume that the intestinal lumen is a rigid cylinder with a constant volume-to-length ratio A and a constant surface-to-volume ratio of $\phi = \Delta S/\Delta V$. Also, assume that chyme flows through the lumen at a constant flow rate Q. The concentrations of polysaccharides C_P and glucose C_G in chyme will be functions of time t and position z along the intestine.

(a) Sketch a control volume consisting of a tube section of length Δz showing convective inputs and outputs of P and G, absorption of G at the tube wall and internal chemical reaction.
(b) Using this control volume, derive the dynamic molar balances for glucose and polysaccharide to describe the concentrations $C_P(z,t)$ and $C_G(z,t)$. Start with $\partial(C_i A \Delta z)/\partial t = ...$ (i=G,P).
(c) By taking the limit of an infinitesimal control volume, derive the differential concentration equations for polysaccharide and glucose. Under what conditions are the same final equations obtained directly from Eq. 2.4-25 in the textbook?

Problem 2-7: Alternative Form of the Concentration Equation

Using Eq. 2.4-26 and then Eqs. 2.4-21 and 2.4-24, show that the species concentration equation given by Eq. 2.4-25 can also be written in the form

$$\frac{\partial C_i}{\partial t} + \frac{Q}{A_t}\frac{\partial C_i}{\partial z} = \frac{C_i}{A_t}\frac{\partial Q_{wall}}{\partial z} - \frac{1}{A_t}\frac{\partial \dot{N}_{i,wall}}{\partial z} + R_i$$

You should begin by expanding the z derivative in Eq. 2.4-25.

Problem 2-8: Different Concentration Units of an Ideal Gas

An ideal gas mixture at P=101 kPa and T=290°K contains equal mass fractions of oxygen and carbon dioxide. Compute the mole fractions, the partial pressures and molar concentrations of the two gases. Also compute the molar density of the ideal gas mixture.

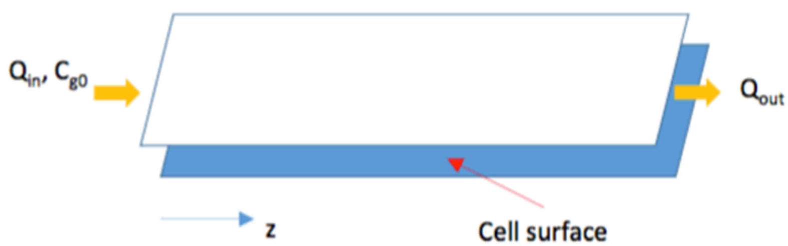
Problem 2-9: Material Balance For a Spatially Lumped System

A bioreactor is used to culture a tissue construct. The construct along with fresh culture medium are initially added to the reactor after which the bioreactor is sealed at a constant volume. As the construct releases a waste product (w) at a rate $\dot{R}_w$ [mol/s], its concentration in the culture medium, C_w[mol/L], continuously increases.

(a) Draw a schematic of the system. Identify the control volume. What are the inputs and outputs?
(b) Starting with Eq. 2.4-1, develop a dynamic model for $C_w(t)$. Justify any simplifying assumptions.
(c) Solve the model equation for $C_w(t)$ considering that $\dot{R}_w$ is constant.

Problem 2-10: Material Balance For a Spatially Distributed System

In a parallel-plate reactor of length L, cells that adhere to the bottom plate are exposed to a constant volumetric inflow Q_{in} of a nutrient medium with constant density ρ. The entering medium contains glucose at a concentration C_{go}. The cells consume glucose at a rate $R_g(z)$ moles/sec-cm^2 of exposed cell surface. Derive a steady-state model for glucose concentration along the reactor $C_g(z)$. Note that the width W of the plates is much greater than their separation distance, h, and the exposed cell surface per unit volume of the reactor, ϕ, is a constant.



(a) Make a drawing of a control volume corresponding to a section Δz beginning at position z along the length of the plates. Label all relevant variables and parameters. How is the volume ΔV and exposed cell surface ΔS in the control volume related to Δz? How is the glucose flux into the exposed cell surface $N_{g,wall}$ related to R_g?
(b) Derive an equation for the volumetric flow Q(z) between the plates from a steady-state solution balance on the control volume assuming the effect of $N_{g,wall}$ is negligible. How does Q vary with z? What is the boundary condition on Q?
(c) Derive an ordinary differential equation for the steady-state glucose concentration $C_g(z)$. What is the boundary condition for C_g?
(d) Solve the model equations for $C_g(z)$ assuming the glucose consumption rate is $R_g(z)=kC_g(z)$ where k is a rate constant.