Connectivity of Large-Scale Cognitive Radio Ad Hoc Networks

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***Abstract*— Connectivity of large-scale wireless networks has received considerable attention in the past several years. Different from traditional wireless networks, in *Cognitive Radio Ad-hoc Networks* (CRAHNs), primary users have spectrum access priority of the licensed bands over secondary users. Therefore, the connectivity of the secondary network is affected by not only the density and transmission power of secondary users, but also the activities of primary users. In addition, the number of licensed bands also has impact on the connectivity of CRAHNs. To capture the dynamic characteristics of opportunistic spectrum access, we introduce the *Cognitive Radio Graph Model* (CRGM) which takes into account the impact of the number of channels and the activities of primary users. Furthermore, we combine the CRGM with continuum percolation model to study the connectivity in the secondary network. We prove that secondary users can form the percolated network when the density of primary users is below the critical density. Then, the upper bound of the critical density of the primary users in the percolated CRAHNs is derived. Simulation results show that both the number of channels and the activities of primary users greatly impact the connectivity of CRAHNs.**

I. Introduction

Connectivity is a challenging issue in large-scale wireless networks. There have been extensive studies on the connectivity of wireless ad hoc networks and sensor networks [1]–[4]. Conventionally, the performance of wireless networks has been examined under the assumption of maintaining full connectivity (or k-connectivity) [2]. However, in CRAHNs [5]–[7], the problem is fundamentally different since the full connectivity criterion is difficult to achieve because of the varying spectrum availability. The *Secondary Users* (SUs) opportunistically utilize the spectrum holes unoccupied by the *Primary Users* (PUs) so that the efficiency of the limited spectrum resource is significantly improved. Therefore, the reachability between two SUs depends not only on the distance between them but also on the availability of the communication channel. As a result, in a secondary network, communication links are time-varying due to the temporal dynamics of spectrum opportunities. When PUs appear, the SUs have to evacuate the borrowed licensed band and move to some other available ones. Some SUs may fail to detect any available channel and have to stop their transmissions until available channels emerge. To investigate the impact of this dynamic behavior on the connectivity of CRAHNs, we present the cognitive radio graph to model the connectivity of different network layers according to the frequency bands. Our model further considers the activities of PUs and the number of channels which significantly affect the connectivity as well.

To address the connectivity problem in CRAHNs, we need to look into the differences between CRAHN and the conventional network. First, the node density impacts the connectivity of the CRAHN in a different way due to the counter effect of the primary network on the secondary network. In the conventional network, a network is connected if the node density is large [8] and vice versa. However, the connectivity of the secondary network depends on the density of PUs in the CRAHN. The secondary network cannot be percolated when there are many active PUs. Second, the connectivity in CRAHNs is subject to temporal variations since PUs could appear at any time without precaution. FCC [9] ruling requires that SU should not incur intolerable interference to the transmission of PUs. In [10], experiments show that even a single packet transmission causes audible interference during the transmission of the wireless microphone. Thus, the activity of PUs affects the connectivity of the secondary network significantly. Third, since SUs can operate in any portion of the licensed spectrum [11], the number of licensed bands also plays an important role in the connectivity of CRAHNs. The wider spectrum the SUs could exploit, the better connectivity the SUs can achieve. Due to these characteristics, it is essential to conduct research on the connectivity in CRAHNs.

Recently, the percolation model, especially the continuum percolation model has been widely used for the analysis of large scale wireless networks [8] [12]. When the nodes are distributed with low density, the network would be partitioned into small fragments. As the density increases, some components emerge where nodes can communicate with one another through single hop or multiple hops. As the density continues to increase, an extremely large connected component forms such that each node in this component can connect to an extremely large number of nodes. A network is considered to be percolated if it contains an extremely large connected component almost surely (a.s.). This phenomenon of a sudden and drastic change from a subcritical phase to a supercritical phase is called a phase transition. However, in CRAHNs, the activities of PUs and the number of channels also lead to phase transition besides the density of PUs and SUs. In this paper, we combine the cognitive radio graph model with continuum

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percolation model to study the connectivity in the secondary network.

The rest of this paper is organized as follows. Section II provides a brief overview of previous works on connectivity. Section III gives the system model. In Section IV, we present our cognitive radio graph model under the dynamic spectrum activities. In Section V, we investigate the connectivity criterion using the percolation theory. Section VI shows the simulation results of the the impact of PU activities and the number of channels on connectivity, followed by the conclusions in Section VII.

II. Related Work

Connectivity is one of the key problems that have been extensively investigated in ad hoc networks in the past decades [13]–[19]. In [13], the authors investigate the impact of interference on the connectivity of ad hoc networks using the random graphs associated with the Poisson Boolean model and percolation properties of these graphs. Dousse et al. study the message delivery latency of wireless sensor networks with uncoordinated power saving mechanisms. They prove that the latency depends only on the network parameters (node density, connectivity range, duration of active and sleeping periods) instead of the random location of the nodes [14]. In [15], Kong and Yeh consider the impact of mobility on the connectivity of the wireless network where the link between two nodes might break when the distance between them increases beyond the transmission range. Zhao et al. give a more precise description of the fundamental relationship between node density and transmission delay in large-scale wireless ad hoc networks with unreliable links [16]. In [17], the authors study the critical phase transition time of large-scale wireless multi-hop networks when the network topology experiences a partition due to increasing random node failures. In [18], the authors utilize the base stations in the large-scale hybrid network to improve the connectivity of ad hoc network in the subcritical phase. Li, Zhang and Fang investigate the critical transmission power to achieve asymptotic connectivity in the network with directional antennas [19].

In CRAHNs, spectrum sharing and resource allocation [20]–[24] have been extensively studied to improve spectrum utilization, while the connectivity remains to be studied. Ren, Zhao and Swami conduct the pioneer study on the connectivity of CR networks [25] [26]. Similar to the works mentioned above, the authors use techniques and theories in continuum percolation to characterize the connected region of the secondary network, then discuss the tradeoff between the proximity (the number of neighbors) and the occurrence of spectrum opportunities. Besides, the analysis of the impact of the transmission power is provided as well. These works, however, only consider the network with a single channel. Furthermore, they do not take into account the time-varying spectrum availability induced by PUs, which is the key characteristic that distinguishes cognitive radio networks from the traditional networks. To investigate the connectivity of CRAHNs, Abbagnale et.al [27] propose a mathematical framework based on the Laplacian spectrum of graphs. The model uses a unique metric, the algebraic connectivity, to capture the network connectivity, the average distance of nodes, and the network diameter. Different to the prior works, we propose a cognitive radio graph model to portray the dynamic features, including the spatial and temporal variation of spectrum, and the number of licensed bands. Then we combine the model with percolation processes to study the dynamic connectivity in large-scale CRAHNs.

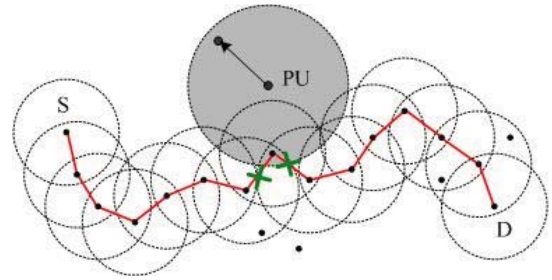


Fig. 1. Impact of frequency diversity on network connectivity

III. System Model

A. Network Model

In this paper, we consider the coexistence of the secondary network with the primary network on plane $\mathcal{R}^2$. Both networks are modeled by the *geometric random graph* [28]. In the primary network, the PUs are distributed according to a two-dimensional Poisson point process with density λ_p. Here, we assume that all PUs have the same transmission range r_p. Each PU is randomly assigned an available channel for transmission by the primary system controller. We model the PU network with a random disk graph $G(\lambda_p, r_p)$. Let λ_s and r_s be the density and the transmission radius of each SU respectively. Similarly, the secondary network is modeled by a random disk graph $G(\lambda_s, r_s)$.

B. Communication Model

In traditional wireless ad hoc networks, the distance between nodes and the transmission power are the critical factors that determine the network connectivity. Two nodes can establish a bi-directional link between them if they are in the transmission range of each other.

However, the connectivity problem in CRAHNs is different from traditional ad hoc networks. SUs experience spectrum heterogeneity because the set of available channels might be quite different from node to node. Moreover, the spectrum availability might change dynamically due to PUs' activities. In CRAHNs, two SUs can connect only if they are in radio visibility and have at least one available common channel. As described in Fig. 1, there is a path from the source node S to the destination node D. If a PU appears on the channel being used by an SU, the SU should abdicate the channel immediately and the suspend the transmission. As a consequence, not only the distance and transmission power, but also the availability of the channel decides the network connectivity in CRAHNs. We introduce some definitions which capture the unique features of the connectivity in CRAHNs in the following.

Definition 1: Geographic Link. A geographic link exists between SU_i and SU_j only if the Euclidean distance between them is less than r_s, i.e., $\|X_i - X_j\| < r_s$.

$G(\lambda_s, r_s)$ models a secondary network without considering the impact of the primary network. It only contains geographic links.

Definition 2: Communication Protection Radius R_p. All the PUs have the same communication protection radius $R_p = (1+\alpha)r_p$, where α is the communication protection coefficient ($\alpha > 0$).

In order to access a frequency band which is used by a PU, the SUs must be located outside the communication protection area of the PU. The protection area is a circle centered at the PU with radius R_p.

Definition 3: Spectrum OPportunities $SOP(SU_i)$. $SOP(SU_i)$ is defined as a set of frequency bands temporarily unoccupied by PUs and available for SU_i.

SU_i can opportunistically access a frequency band in $SOP(SU_i)$, but it has to immediately vacate this band when an PU becomes active on the frequency band.

Definition 4: Radio Link. A radio link exists between SU_i and SU_j if they have common available channels, i.e., $SOP(SU_i) \bigcap SOP(SU_j) \neq \phi$.

Definition 5: Communication Link. We say SU_i and SU_j have a communication link if there exist both geographic link and radio link between them.

The existence of a communication link between two secondary users is determined by the node distance and the availability of the spectrum opportunity at both nodes.

C. Preliminary of Percolation

Mathematically, we introduce the following percolation terminologies to further understand how the continuum percolation process on a geometric random graph is related to the connectivity of a large-scale cognitive radio network.

Definition 6: Giant component C_{max} is the largest connected subgraph in graph $G(\lambda_s, r_s)$.

Definition 7: Percolation probability $Prob_\infty(\lambda_s)$ is the probability that the giant component of graph $G(\lambda_s, r_s)$ has an infinite number of secondary nodes, i.e., $Prob_\infty(\lambda_s) = Prob(|C_{max}| = +\infty)$.

Definition 8: Critical density. In CRAHNs, both PUs and SUs have critical density. The critical density of PUs λ_p^c is the maximum node density that can make SUs form a percolated secondary network. By contrast, the critical density of SUs λ_s^c is the minimal node density that can ensure the percolation probability $Prob_\infty(\lambda_s)$ of graph $G(\lambda_s, r_s)$ to be larger than 0, i.e., $\lambda_s^c = \ inf\{\lambda_s : \ Prob_\infty(\lambda_s) > 0\}$. The λ_p^c and λ_s^c pair which defines the connectivity of the CRAHNs is to be investigated in this paper.

IV. Cognitive Radio Graph Model

In order to describe the impact of the number of channels and the activities of PUs, we introduce a cognitive radio graph

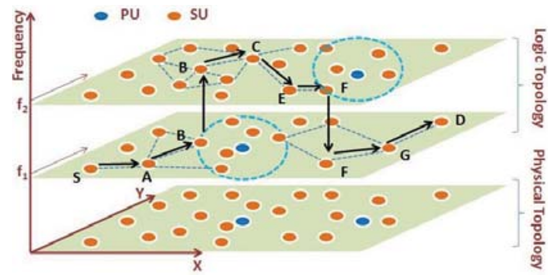


Fig. 2. Cognitive radio graph model

model to extend our network model $G(\lambda_s, r_s)$ defined above.

Note that more free channels result in higher degree of link connectivity. Since the existence of a communication link between two nodes depends on the availability of the spectrum allocated to the geographic link, the connectivity of a cognitive radio network depends on the available spectrum pool. Let E_k denote the set of radio links in the network corresponding to channel k. G_k is the graph corresponding to channel k, denoted $G_k = (V, E_k)$. A link $l = (v_i, v_j) \in E_k$ iff v_j is within the transmission range of v_i when the radio link is assigned an available channel k. According to the definition of geographic link and radio link, the graph $G(\lambda_s, r_s)$ has both physical topology and logic topology as shown in Fig. 2. Each layer of the logic topology is a subgraph G_k showing the connectivity in the network corresponding to a particular licensed channel k. We now define $G = (V, E)$ as our cognitive radio graph model, where E denotes the set of all possible radio links, i.e., $E = \bigcup_{k=1}^{N} E_k$. G is therefore a multi-layer graph. Provided that all nodes have the same transmission power, the connectivity in each subgraph is determined by the density and activity of SUs and PUs.

In order to explore the impact of the activity of PUs and the number of channels in CRGM, we model the process of the licensed spectrum access as a continuous-time Markov chain. Assume there are a total of N licensed channels. Each PU uses only one channel to transmit. When a PU comes, it can claim the channel being used by SUs. In the Markov chain model, the state in the transition diagram shown in Fig. 3 is described by the number of available channels for SUs, denoted i. At time t, the state space $S(t)$ is given by

$$S(t) = \{i | 0 \leq i \leq N, t \geq 0\}.$$

The arrival of PUs is modeled as a Poisson process with arrival rate γ_p. The service time of PUs follows negative exponential distribution with expectation $1/\mu_p$.

We denote $P_i(t)$ as the steady-state probability distribution of state $i \in S(t)$ at time t. The state i can be translated into one of the following states depending on the arrival and departure of PUs. State $i-1$ will be reached if a new PU arrives and operates in a licensed channel with probability γ_p. State $i+1$ can be reached if the PU finishes the transmission with probability $(N-i)\mu_p$.

We can get the steady-state probabilities by solving the above model. In the solution, the probability that the SU has

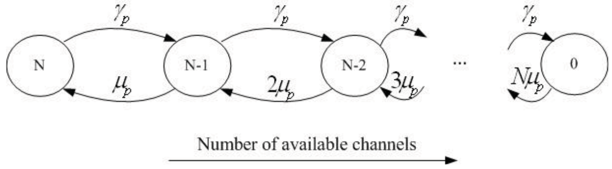


Fig. 3. The continuous-time Markov chain model

communication links plays a significant role in the connectivity of CRAHNs. We define this probability as the survival function of SU which will be used extensively in the following parts.

Definition 9: Survival Function $s(t)$. The survival function $s(t)$ actually serves as the probability that an SU has at least one communication link at time t. $s(t)$ is determined by the number of licensed channels as well as the activity of PUs because communication links are time-varying according to the temporal dynamics of SOP.

Assume that the number of the available channels is i for SU_1 and j for the adjacent node SU_2. Then, the probability that there exists at least one common channel between SU_1 and SU_2 is

$$P_c = \begin{cases} 1 & if \ i+j > N, \\ 1 - \frac{C_{N-i}^{j}}{C_N^j} & if \ i+j \leq N. \end{cases}$$

Then, we can derive the survival function of SU as

$$s(t) = \sum_{i=0}^{N} P_i(t) Q_i(t),$$

where $Q_i(t)$ indicates the probability that SU_1 has common channels with other adjacent nodes at state i. $Q_i(t)$ can be calculated with

$$Q_i(t) = \sum_{j=0}^{N-i} P_j(t)(1 - \frac{C_{N-i}^{j}}{C_N^j}) + \sum_{j=N-i+1}^{N} P_j(t).$$

V. Dynamic connectivity of CRAHNs

In this section, we study the connectivity in a secondary network $G(\lambda_s, r_s)$ which coexists with the primary network $G(\lambda_p, r_p)$. Since the random distribution and activities of PUs will affect the set of available channels for SUs, the communication link between SUs changes over location and time. To capture this characteristic, we combine CRGM defined in the above section with the dynamic percolation process to study the connectivity in the secondary network. In CRGM, each node is associated with a survival function $s(t)$. According to the Thinning theorem [29], the point process of surviving users is also a Poisson process with density function $\lambda(t) = s(t)\lambda_s$. Let $G(\lambda_s, r_s, s(t))$ denote the sampled secondary network at time t. Therefore, $G(\lambda_s, r_s, s(t))$ only comprises the SUs in $G(\lambda_s, r_s)$ that have communication links.

A. Mapping

On the continuous plane $\mathcal{R}^2$, we construct a discrete square lattice $\mathcal{L}$ with edge length d. Thus, the percolation process on $\mathcal{R}^2$ can be mapped into a bond percolation on $\mathcal{L}$, or the existence of an infinite path made of open edges.

As shown in Fig.4, for a horizontal edge AB, define the associated rectangle $S_{AB} = [a_x d - \frac{1}{2}d, a_x d + \frac{3}{2}d] \times [a_y d - \frac{1}{2}d, a_y d + \frac{1}{2}d]$, where $(a_x d, a_y d)$ and $(a_x d + d, a_y d)$ are the coordinates of the two end vertices.

Definition 10: Let E_{AB} denote the event that an edge AB is open. Then E_{AB} occurs if SUs in the rectangle satisfy the following conditions.

- There is a sequence $S_1, S_2, S_3 \cdots S_l$ across the square rectangle $S_{AB} = [a_x d - \frac{1}{2}d, a_x d + \frac{3}{2}d] \times [a_y d - \frac{1}{2}d, a_y d + \frac{1}{2}d]$ from left to right.
- There is a sequence $S_1, S_2, S_3 \cdots S_m$ across the square rectangle $S_{AB}^{+} = [a_x d - \frac{1}{2}d, a_x d + \frac{1}{2}d] \times [a_y d - \frac{1}{2}d, a_y d + \frac{1}{2}d]$ from top to bottom.
- There is a sequence $S_1, S_2, S_3, \cdots S_n$ across the square rectangle $S_{AB}^{-} = [a_x d + \frac{1}{2}d, a_x d + \frac{3}{2}d] \times [a_y d - \frac{1}{2}d, a_y d + \frac{1}{2}d]$ from top to bottom.
- Two adjacent nodes contained in the sequence have Euclidean distance $\| X_{S_i} - X_{S_{i+1}} \| \leq r_s$, where X_{S_i} and $X_{S_{i+1}}$ are the x-coordinates of nodes S_i and S_{i+1} respectively.
- Two adjacent nodes contained in the sequence have a communication link.

Note that the open vertical edges of $\mathcal{L}$ can be defined similarly by rotating the rectangles by 90 degrees. The dual lattice of $\mathcal{L}$, which is denoted by $\mathcal{L}'$, is obtained by putting the vertices in the center of each square of $\mathcal{L}$, and then joining two such neighboring vertices by a line across an edge of $\mathcal{L}$. Thus, the dual lattice $\mathcal{L}'$ is a square lattice which is shifted by $(d/2, d/2)$ from lattice $\mathcal{L}$. An edge of $\mathcal{L}$ is said to be open if and only if its corresponding edge of $\mathcal{L}'$ is open.

B. Connectivity under Dynamic Spectrum Activity

Given the mapping and open edge defined above, we now show the relationship between the open edge and the secondary network connectivity.

Lemma 1: If lattice $\mathcal{L}$ is percolated, then the secondary network $G(\lambda_s, r_s, s(t))$ has an infinite connected component on the continuous plane $\mathcal{R}^2$.

Proof: The presence of bond percolation on $\mathcal{L}$ means that there exists an infinite open path consisting of open edges on $\mathcal{L}'$. Thus, the basic idea of the proof for Lemma 1 is to translate the presence of continuum percolation on $G(\lambda_s, r_s, s(t))$ into the existence of an infinite open path on $\mathcal{L}'$.

To prove Lemma 1, we consider an infinite open path on $\mathcal{L}'$. For an edge over the path, the vertex is located at the center of a square of $\mathcal{L}$. Therefore, along the edge, there exist two adjacent squares that satisfy the conditions given in Definition 10. The last two conditions guarantee that each SU covered by two adjacent squares along the open edges can communicate with each other through a single hop or multiple hops. For any two adjacent edges, their associated rectangles

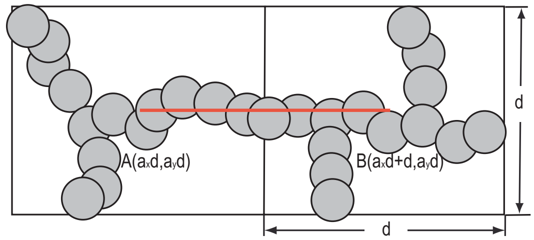


Fig. 4. Open rectangle

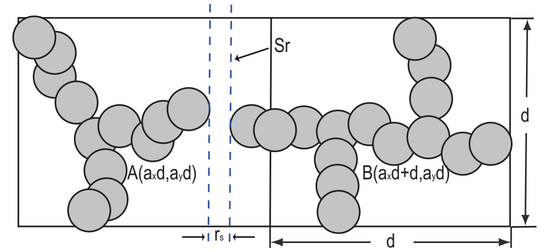


Fig. 5. Vacant band

intersect in a same square of $\mathcal{L}$. Since both of the two edges are all open, there exists a connected component crossing the two rectangles associated with the edges. In addition, the open path is assumed infinite. The squares of all open edges on $\mathcal{L}$ are also infinite which form an infinite connected component in $G(\lambda_s, r_s, s(t))$. Therefore, an infinite open path on $\mathcal{L}'$ implies an infinite connected component in $G(\lambda_s, r_s, s(t))$. ■

Theorem 1: If $\lambda_p > \lambda_p^c$, with probability one there exists no infinite connected component in $G(\lambda_s, r_s, s(t))$ for all times $t > 0$.

Proof: Under a Poisson Boolean model, the whole space is partitioned into two regions, the occupied region, which is the region covered by at least one node, and the vacant region, which is the complement of the occupied region. We define occupied (vacant) components as those connected components in the occupied (vacant) region. Therefore, the connectivity of CRAHNs can be studied through examining the occupied connected components in the corresponding Poisson Boolean model $G(\lambda_s, r_s, s(t))$. For the Poisson Boolean model, phase transition appears more remarkably in the sense that the critical density for the a.s. existence of infinite occupied components is equal to that for the a.s. inexistence of infinite vacant components. Now we prove that when the density of PUs exceeds a threshold, the vacant component left to the secondary network is not enough to construct an infinite connected component.

We randomly select a vacant component V in the following way. First, randomly choose a point υ on the plane $\mathcal{R}^2$. Denote the vacant component belongs to υ as $V\{\upsilon\}$. If υ is not covered by any node, then let $V = V\{\upsilon\}$. Otherwise, it goes back to the first step. Therefore, a large vacant component is more likely to be chosen. According to Lemma 4.1 in [29], if $\lambda_p > \lambda_p^c$, then

$$P(\sigma(V\{\upsilon\}) \geq a) \leq \alpha e^{-\beta a},$$

where σ is the diameter of V, and α, β are constants with $\alpha < \infty$ and $\beta > 0$.

Recall how V is selected, we have

$$P(\sigma(V) \geq a) = \frac{P(\sigma(V\{\upsilon\}) \geq a)}{P_s} \leq \alpha' e^{-\beta' a},$$

where P_s is the proportion of space not covered by nodes, which is a constant when the node density is given. α',β' are constants with $\alpha' < \infty$ and $\beta' > 0$. Let V_s be the selected connected component consisted of SUs which is less than the vacant component. Then, we can get

$$\begin{aligned} \lim_{a\to\infty} P(\sigma(V_s) \geq a) &\leq \lim_{a\to\infty} P(\sigma(V) \geq a) \\ &\leq \lim_{a\to\infty} \alpha' e^{-\beta' a} \\ &= 0. \end{aligned}$$

Therefore, when $\lambda_p > \lambda_p^c$, an unbounded occupied component of PUs exists with probability one. Thus, there exists no infinite connected component in $G(\lambda_s, r_s, s(t))$ for all times $t > 0$. ■

Theorem 2: Given secondary network $G(\lambda_s, r_s, s(t))$ coexisting with primary network $G(\lambda_p, r_p)$, there exists a critical density $s(t)\lambda_s > \lambda_s^c$ and $\lambda_p < \lambda_p^c$. Then with probability one, there exists an infinite connected component in the secondary network $G(\lambda_s, r_s, s(t))$ for all times $t > 0$.

Proof: As shown in Fig.5, suppose that there is a band with width r_s crossing vertically through S_{AB}, then the intersection of the band and S_{AB} forms a rectangular with length d and width r_s, denoted by S_r. Let

$$S_r^c = \{No\ surviving\ secondary\ nodes\ located\ in\ S_r\}.$$

Let $E_{AB}^{\sim}$, E_{AB}^{+} and E_{AB}^{-} be the events that rectangles S_{AB}, S_{AB}^{+} and S_{AB}^{-} are open, respectively. Then, $E_{AB}^{\sim}$ occurs when S_r^c occurs. We can also get the bands crossing horizontally through S_{AB}^{+} and S_{AB}^{-}. Assume the positions of the secondary nodes are independently and identically distributed (i.i.d.). Then there exists

$$P_r(E_{AB}^{\sim}) = P_r(E_{AB}^{+}) = P_r(E_{AB}^{-}) = 1 - P_r(S_r^c).$$

As aforementioned, the point process of surviving secondary nodes is a Poisson process with density function $\lambda(t) = s(t)\lambda_s$, then we have

$$P_r(S_r^c) = exp(-\lambda_s r_s d s(t)),$$

where d is the edge width with tight bound $\Theta(\sqrt{\log n})$ [17].

Note that E_{AB} occurs if the conditions given in Definition 10 are satisfied. Since only the surviving SUs are included in $G(\lambda_s, r_s, s(t))$, E_{AB} occurs if the first three conditions are satisfied in $G(\lambda_s, r_s, s(t))$. Utilizing Fortuin-Kasteleyn-Ginibre (FKG) inequality in [29] [30], the probability that E_{AB} occurs is lower bounded by

$$\begin{aligned} P_r(E_{AB}) &= P_r(E_{AB}^{\sim} \bigcap E_{AB}^{+} \bigcap E_{AB}^{-}) \\ &\geq P_r(E_{AB}^{\sim}) P_r(E_{AB}^{+}) P_r(E_{AB}^{-}) \\ &= (1 - P_r(S_r^c))^3. \end{aligned}$$

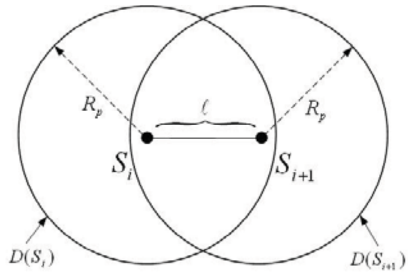


Fig. 6. A link between S_i and S_{i+1}

Now, consider a path $\mathcal{P}_m = \{e_i\}_{i=1}^m$ of length m in $\mathcal{L}$. Let E_{e_i} denote the event that e_i is open. Assume all the events in $\{E_{e_i}\}_{i=1}^m$ are independent. Then, the probability that the path $\mathcal{P}_m$ is open can be

$$\begin{aligned} P_r(\mathcal{P}_m \ open) &= P_r(\textstyle\bigcap_{i=1}^m E_{e_i}) \\ &\geq \textstyle\prod_{i=1}^m P_r(E_{e_i}) \\ &= (1 - P_r(S_r^c))^{3m}. \end{aligned}$$

As a consequence, the probability that there exists an infinite open path starting from the origin on $\mathcal{L}'$ is arbitrarily close to 1 by choosing large enough edge width d when the continuum model is in the supercritical phase. From Lemma 1, the existence of an infinite path on $\mathcal{L}'$ further implies the existence of an infinite connected component in $G(\lambda_s, r_s, s(t))$ when $s(t)\lambda_s > \lambda_s^c$. However, Theorem 1 implies that if $G(\lambda_s, r_s, s(t))$ percolates, λ_p is less than λ_p^c. Therefore, if $s(t)\lambda_s > \lambda_s^c$ and $\lambda_p < \lambda_p^c$, there exists an infinite connected component with probability one in the secondary network $G(\lambda_s, r_s, s(t))$ for all times $t > 0$.

■

As we can see, for any given network size, $P_r(E_{AB})$ increases exponentially as the survival function $s(t)$ increases. Specifically, $P_r(E_{AB})$ goes to 0 when $s(t) \to 0$. This is in accordance with the fact that the more failed nodes, the more difficult to have a connected component in the graph.

Theorem 3: The upper bound on the critical density λ_p of PUs is given by

$$\lambda_p^c = \frac{N \log(1 - \sqrt[N]{1 - \frac{\lambda_s^c}{\lambda_s}})}{2R_p^2(\pi - \arccos \frac{r_s}{2R_p}) + r_s\sqrt{2R_p^2 - \frac{r_s^2}{4}}}.$$

Proof: For every pair of SUs in an infinite connected component, there exist available channels between the transmitter and the receiver. Fig. 6 shows a link between two adjacent SUs S_i and S_{i+1}. ℓ is the distance between S_i and S_{i+1}. The two SUs cover circle areas $D(S_i)$ and $D(S_{i+1})$ with radius R_p respectively. Let $A(S_i, S_{i+1}) = D(S_i) \bigcup D(S_{i+1})$, so

$$A(S_i, S_{i+1}) = 2R_p^2(\pi - \arccos \frac{\ell}{2R_p}) + \ell\sqrt{2R_p^2 - \frac{\ell^2}{4}}.$$

A set Φ_k is defined as the set which contains the PUs using channel k. Assume a PU appears on a channel randomly and independently. Φ_k can be obtained with a random sampling process in which a PU is selected with probability $\frac{1}{N}$. According to the Thinning theorem [29], Φ_k is a Poisson process with parameter $\frac{\lambda_p}{N}$. Similarly, the sets $\Phi_1, \Phi_2, \cdots, \Phi_k$ are all Poisson processes and independent of each other. Then, we define $\Phi_k(A)$ as the set of PUs in area $A(S_i, S_{i+1})$ using channel k. Therefore, the number of PUs in $\Phi_k(A)$, denoted as $|\Phi_k(A)|$, follows a Poisson distribution with parameter $\frac{\lambda_p A(S_i, S_{i+1})}{N}$. Now we can get the probability that no PU occupies the channel k

$$P(|\Phi_k(A)| = 0) = \exp(\frac{-\lambda_p A(S_i, S_{i+1})}{N}).$$

So, the probability that there exists at least one available channel in area $A(S_i, S_{i+1})$ can be written as

$$P_A = 1 - (1 - \exp(\frac{-\lambda_p A(S_i, S_{i+1})}{N}))^N.$$

According to the Thinning theorem [29], $P_A\lambda_s > \lambda_s^c$ should be satisfied in an infinite connected component. Therefore,

$$\lambda_p < \frac{N \log(1 - \sqrt[N]{1 - \frac{\lambda_s^c}{\lambda_s}})}{A(S_i, S_{i+1})}.$$

For an infinite open path, the distance between two adjacent nodes should be no more than r_s. Therefore,

$$A(S_i, S_{i+1}) \leq 2R_p^2(\pi - \arccos \frac{r_s}{2R_p}) + r_s\sqrt{2R_p^2 - \frac{r_s^2}{4}}.$$

Thus,

$$\begin{aligned} \lambda_p &< \lambda_p^c \\ &= \frac{N \log(1 - \sqrt[N]{1 - \frac{\lambda_s^c}{\lambda_s}})}{2R_p^2(\pi - \arccos \frac{r_s}{2R_p}) + r_s\sqrt{2R_p^2 - \frac{r_s^2}{4}}}. \end{aligned}$$

■

VI. Simulation Results

We are interested in the relationship between the connectivity and the dynamic characteristics of spectrum which include the activities of PUs and the number of channels. More specifically, the effects of arrival rate and service time of PUs on the connectivity are investigated through extensive simulation. We use survival probability to show the connectivity of the secondary network. The survival probability is the value of the survival function defined in Section III at time t_0. The higher the survival probability, the better the connectivity can be achieved. In addition, a drastic change of the survival probability may lead to the phase transition of connectivity.

In our simulation, we assume the arrival of the primary nodes follows a Poisson process. The channel service time of the PUs is exponentially distributed with rate μ_p.

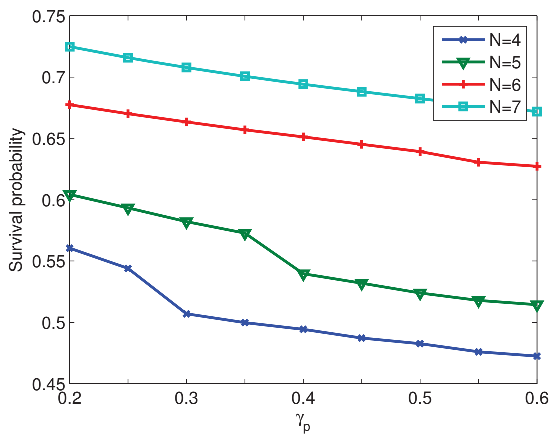


Fig. 7. SU survival probability with γ_p

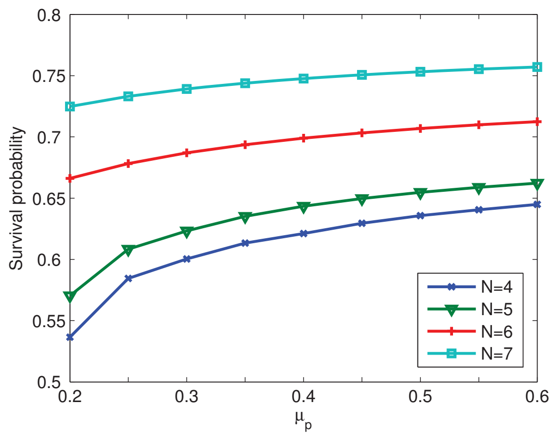


Fig. 8. SU survival probability with μ_p

A. Impact of Arrival Rate of PUs

In this section, we study the impact of the arrival rate of PUs on the survival function. The number of channels, denoted by N, varies from 4 to 7. The arrival rate of the PUs on these channels follows a Poisson process with parameter $\gamma_p = (0.2 \sim 0.6)$. The service rate of PU is $\mu_p = 0.4$.

As shown in Fig. 7, the survival probability for a particular N decreases with the increased PU arrival rate. With more PU arrivals, more SUs would be blocked since more channels are occupied by PUs. In addition, frequent PU arrivals prevent the transmission of SUs from being completed. Observe that the survival probability when $N = 7$ outperforms the other three as the arrival rate rises. This demonstrates that a bigger channel pool helps improve the survival probability of the SUs. In addition, there are drastic changes when γ_p is set to 0.25 and 0.35 for $N = 4$ and $N = 5$ respectively. These changes lead to phase transitions of connectivity which will be illustrated in subsection VI-C.2. As N increases to 6 and 7, more spectrum opportunities are exploited, so the impact of the arrival rate on the survival probability declines. In the case of large N, the drastic change does not occur as γ_p grows.

B. Impact of Service Time of PUs

In this section, we study the impact of the service time of PUs on the survival function. The channel service rate of the primary nodes is exponentially distributed with parameter μ_p which varies from 0.2 to 0.6 with an interval of 0.05. The arrival rate of PUs is set to 0.1.

Observed from Fig. 8, as μ_p grows, the survival probability increases. By increasing μ_p, the average channel holding time of PU is actually decreasing. Therefore, the survival probability increases since the spectrum is relatively less crowed. Similarly to Fig. 7, the survival probability increases as the number of channels N rises given certain μ_p. Drastic changes happen at $N = 4$ and $N = 5$ when μ_p is set to 0.25. The changes trigger phase transitions too. We also give an example of the connectivity at $N = 5$ in subsection VI-C.3.

C. Examples of Connectivity in CRAHNs

In this section, we investigate the phase transition in accordance with the change of survival probability with some examples. The previous two subsections show that the survival probability is related with the number of channels, the arrival rate and the service time of PUs. Therefore, we study the phase transition phenomenon with respect to these three aspects.

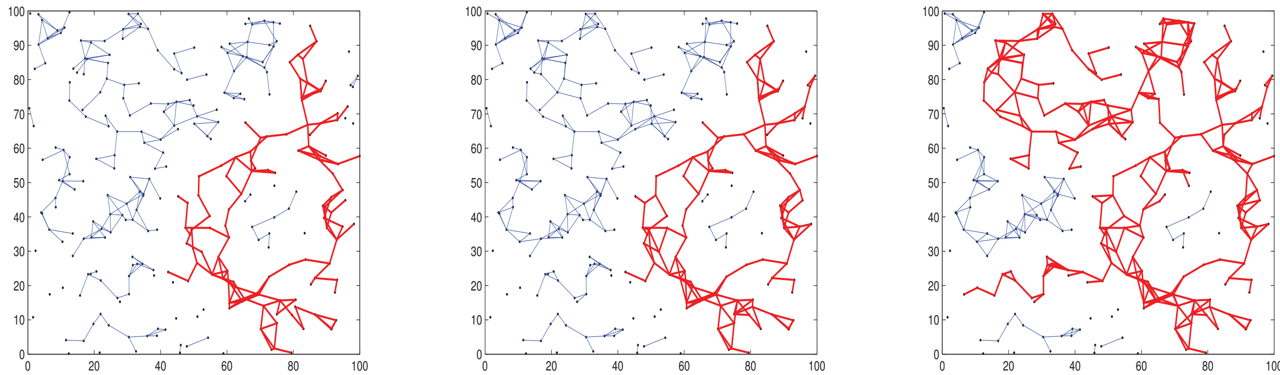
Without loss of generality, we only consider the situation that $\lambda_p < \lambda_p^c$, which means there could be a giant component including infinite secondary nodes. In order to measure the extent of the giant component C_{max}, we introduce the ratio of the giant component to the whole component, given as

$$\rho_g = \frac{number\ of\ secondary\ nodes\ in\ C_{max}}{number\ of\ secondary\ nodes\ in\ the\ whole\ network}.$$

The simulation results are shown in Fig. 9-11.

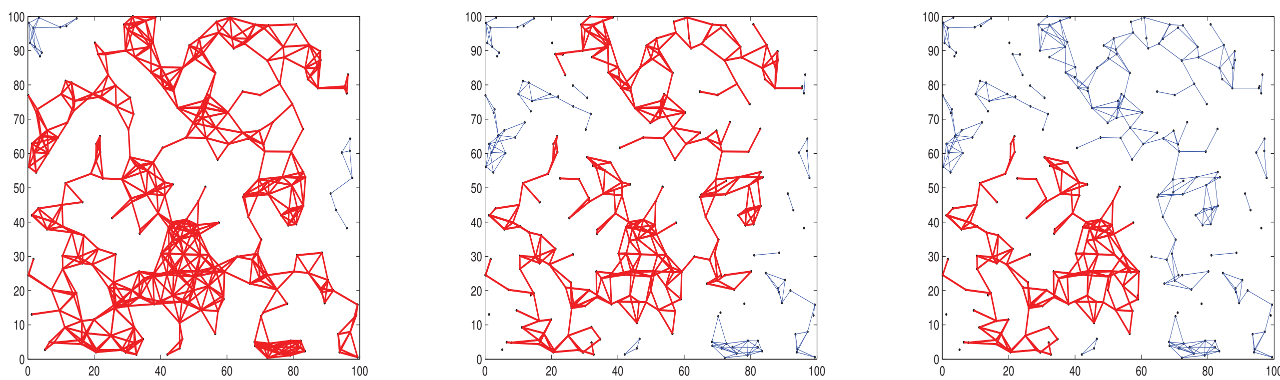
1) Examples of phase transition according to N: Fig. 9 (a), (b), (c) show the examples of graph $G(\lambda_s, r_s, s(t))$ with $N = 4, 5, 6$, respectively. The other parameters are $\gamma_p = 0.2,\ \mu_p = 0.4$. According to the communication link defined in Definition 5, there is at least one common available channel on the links in Fig. 9. The links colored red belong to the giant component. From Fig. 9 (a), we can see that the graph is split into many components with $\rho_g = 35\%$. In Fig. 9 (a) and (b), when the number of channels is increased from 4 to 5, ρ_g grows slightly. However, when N reaches 6, a giant component containing a large portion ($\rho_g = 72\%$) of nodes emerges. This indicates that a phase transition happens at the critical point of $N = 6$.

2) Examples of phase transition according to γ_p: Fig. 10 (a), (b), (c) show the examples of graph $G(\lambda_s, r_s, s(t))$ with $\gamma_p = 0,\ 0.35,\ 0.4$, respectively. The other parameters are $N = 5,\ \mu_p = 0.4$. From Fig. 10 (a), we can see that the graph contains a giant component with $\rho_g = 94\%$. Since γ_p is set to 0, there is no PUs arrival. So the giant component contains all the geographic links. In Fig. 10 (b), when γ_p increases to 0.35, ρ_g reduces to 73%. Fig. 10 (c) shows that ρ_g drops to 38% drastically when γ_p continues increasing to 0.4. This shows a phase transition at the critical point of $\gamma_p = 0.4$.



(a) $N=4,\ \rho_g=35\%$ (b) $N=5,\ \rho_g=37\%$ (c) $N=6,\ \rho_g=72\%$

Fig. 9. An example of phase transition according to the number of channels N ($\gamma_p=0.2,\ \mu_p=0.4$). (When N grows from 4 to 5, ρ_g increases slightly (from 35% to 37%, shown in (a) and (b)). When N increases to 6, a giant component $\rho_g=72\%$) is formed (shown in (c)).)



(a) $\gamma_p=0,\ \rho_g=94\%$ (b) $\gamma_p=0.35,\ \rho_g=73\%$ (c) $\gamma_p=0.4,\ \rho_g=38\%$

Fig. 10. An example of phase transition according to the arrival rate of PUs γ_p ($N=5,\ \mu_p=0.4$). (At first, there is no PU arrival, so a giant component ($\rho_g=94\%$) exists (shown in (a)). When γ_p grows to 0.35, ρ_g reduces to 73% (shown in (b)). However, when γ_p increases to 0.4, ρ_g drastically decreases to 38% (shown in (c)).)

3) Examples of phase transition according μ_p: Fig. 11 (a), (b), (c) show examples of graph $G(\lambda_s, r_s, s(t))$ with μ_p set to $0.1,\ 0.2,\ 0.25$, respectively. The other parameters are $N=5,\ \gamma_p=0.1$. In Fig. 11 (a), the graph is split into many components with $\rho_g=42\%$. When μ_p increases from 0.1 to 0.2, ρ_g grows slightly. However, when μ_p increases to 0.25, ρ_g increases to 86% drastically, thus a giant component is formed. This shows that a phase transition happens at the critical point of $\mu_p=0.25$.

VII. Conclusion

The connectivity of large-scale cognitive radio networks has become an important yet challenging issue. Existing works on connectivity mainly focus on the conventional connectivity problem of the wireless network and do not consider the spectrum dynamics. So they are not suitable for cognitive radio networks. To exploit the specific characteristics, we propose a cognitive radio graph model which introduces survival probability to measure the impact of the number of channels and the activity of primary users. Using theories and techniques from continuum percolation, we investigate the dynamic connectivity and derive the critical conditions to ensure connectivity in the CRAHN. Finally, we demonstrate the simulation results.

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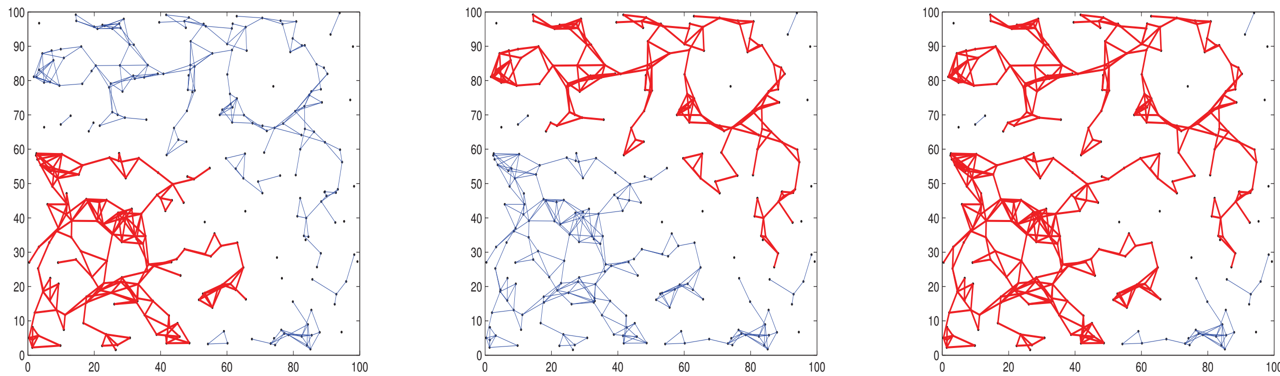

(a) $\mu_p = 0.1,\ \rho_g = 42\%$

(b) $\mu_p = 0.2,\ \rho_g = 45\%$

(c) $\mu_p = 0.25,\ \rho_g = 86\%$

Fig. 11. An example of phase transition according to μ_p ($\gamma_p = 0.2, \mu_p = 0.4$). (At first, the network is partitioned into small fragments (shown in (a)). When μ_p rises from 0.1 to 0.2, ρ_g changes little (from 42% to 45%, shown in (b)). However, when μ_p increases to 0.25, ρ_g increases to 86% suddenly (shown in (c)).)

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