The Capacity of Heterogeneous Wireless Networks

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***Abstract*—Although capacity has been extensively studied in wireless networks, most of the results are for homogeneous wireless networks where all nodes are assumed identical. In this paper, we investigate the capacity of heterogeneous wireless networks with general network settings. Specifically, we consider a dense network with n *normal nodes* and $m = n^b$ $(0 < b < 1)$ more powerful *helping nodes* in a rectangular area with width $b(n)$ and length $1/b(n)$, where $b(n) = n^w$ and $-\frac{1}{2} < w \leq 0$. We assume there are n flows in the network. All the n normal nodes are sources while only randomly chosen n^d $(0 < d < 1)$ normal nodes are destinations. We further assume the n normal nodes are uniformly and independently distributed, while the m helping nodes are either regularly placed or uniformly and independently distributed, resulting in two different kinds of networks called *Regular Heterogeneous Wireless Networks* and *Random Heterogeneous Wireless Networks*, respectively. In this paper, we attempt to find out what a heterogeneous wireless network with general network settings can do by deriving a lower bound on the capacity. We also explore the conditions under which heterogeneous wireless networks can provide throughput higher than traditional homogeneous wireless networks.**

I. Introduction

Capacity is one of the most important issues in wireless networks. Gupta and Kumar [11] show that the per-node throughput capacity is $\Theta(1/\sqrt{n \log n})$[1] bits per second in random ad hoc networks, and the per-node transport capacity is $\Theta(1/\sqrt{n})$ bit-meters per second in arbitrary ad hoc networks. Following this work, extensive research on capacity has been conducted in both static ad hoc networks such as [4], [6]–[8], [29], and in mobile ad hoc networks such as [3], [9], [10], [16], [20], [26], [28].

However, all the works above are for homogeneous wireless networks where all nodes are assumed identical. While in practice, there are many applications in which wireless networks are heterogeneous. For example, in a city-wide wireless network, those wireless devices mounted on top of the buildings are usually much more powerful than normal network users. To give another example, in a wireless network deployed in battlefields, many military vehicles like tanks and planes are much more powerful than normal soldiers, and the communications between them may even be carried out on a higher frequency and have much larger bandwidth.

Some researchers propose to connect the more powerful nodes with a wired network, resulting in the so-called "*hybrid wireless networks*" [1], [13], [15], [19], [21], [35], [36]. However, this approach has some shortcomings. First, it is cost-expensive since the powerful nodes they use are in fact base stations and an optical network connecting the base stations has to be established first. Second, it is also time-consuming because it is not so easy and may take a lot of time to set up such an optical network. Third, current results show that we can achieve throughput higher than that in traditional static homogeneous ad hoc networks by an exponential order only when the number of base stations is $\Omega(\sqrt{n})$ [21], which makes this approach more expensive. Moreover, in some cases the base stations and wired networks are even unavailable. For example, in the rescue affairs after natural disasters like earthquakes, the existing infrastructures may have been damaged and we need to set up a network without them immediately.

Thus, here we consider *heterogeneous wireless networks*, in which all the transmissions are carried out via wireless medium, and there are normal nodes and some more powerful helping nodes. In heterogeneous wireless networks, MAC protocol design is studied by [17], [18], [27], [37], routing protocol design is studied by [2], [23]–[25], [33], [34], and topology control is studied by [14], respectively. However, the capacity problem has not been well studied before.

In this paper, we investigate the throughput capacity of heterogeneous wireless networks. We want to find out that instead of placing base stations interconnected by a wired network, how we can improve the capacity of homogeneous ad hoc networks by deploying some more powerful wireless helping nodes. Notice that most of the previous research on capacity assumes the network area is a square, and the traffic is symmetric implying that the number of source nodes is the same as the number of destination nodes. However, those are only special cases. In practice, the shape of a network area is determined by the distribution of network users, which is further dependent on many factors such as geographical characteristics. For example, about 75% of the population of Utah lies in a corridor which stretches approximately from Brigham City at the north end to Nephi at the south end

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[1]In this paper, we use the following notations [12]: $f(n) = \mathrm{O}(g(n))$ means $f(n)$ is asymptotically upper bounded by $g(n)$; $f(n) = \Omega(g(n))$ means $f(n)$ is asymptotically lower bounded by $g(n)$; $f(n) = \Theta(g(n))$ means $f(n)$ is asymptotically tight bounded by $g(n)$.

along the Wasatch Mountains [32]. So, instead of a square, we may consider the network area in rectangle. Besides, in many applications such as peer-to-peer (P2P) networking, the number of source nodes is usually different from that of destination nodes. Thus, instead of symmetric traffic, we consider asymmetric traffic. In the literature, Liu et al. [22] study the capacity of 2-dimensional strip hybrid wireless networks with symmetric traffic, and Toumpis [31] studies the throughput capacity of 2-dimensional square ad hoc networks with asymmetric traffic. Li et al. [15] investigate the impacts of both network area shape and traffic pattern on the throughput capacity of hybrid wireless networks. But, when it comes to heterogeneous wireless networks, the situation becomes more complicated and it is really worthwhile to seriously analyze the capacity of the networks again. Besides, [15] considers an extended network, while we consider a dense network here as we will introduce soon.

Specifically, in this paper, we consider a dense network with n *normal nodes* and $m = n^b$ $(0 < b < 1)$ more powerful *helping nodes* in a rectangular area with width $b(n)$ and length $1/b(n)$, where $b(n) = n^w$ and $-\frac{1}{2} < w \leq 0$. We consider there are n flows in the network. All the n normal nodes are sources while only n^d $(0 < d < 1)$ randomly chosen normal nodes are destinations. Helping nodes do not serve as data sources or data destinations. Instead, they only help relay packets for the normal nodes. Moreover, notice that in real wireless networks, the normal nodes are usually WiFi users which have low data rates while the helping nodes may use more advanced technologies such as MIMO (Multi-Input and Multi-Output) and UWB (Ultra-WideBand), which can provide much higher data rates. Without loss of generality, we assume the normal users share a bandwidth of 1 and the helping nodes have much higher bandwidth for the transmissions between themselves. We further assume the n normal nodes are uniformly and independently distributed, while the m helping nodes are either regularly placed or uniformly and independently distributed, resulting in two different kinds of networks called *Regular Heterogeneous Wireless Networks* and *Random Heterogeneous Wireless Networks*, respectively. We attempt to find out what a heterogeneous wireless network with general network settings can do by deriving a lower bound on the capacity. We also explore the conditions under which heterogeneous wireless networks can provide throughput higher than traditional pure static ad hoc wireless networks by an exponential order.

The rest of this paper is organized as follows. Section II gives the heterogeneous wireless network model, including topology model, traffic model, and achievable transmission rate model. In Section III and Section IV, we derive a lower bound on throughput capacity of heterogeneous wireless networks, when helping nodes are regularly and randomly distributed, respectively. We finally conclude this paper in Section V.

II. Heterogeneous Wireless Network Model

In this section, we introduce the topology model, traffic model, and achievable transmission rate model for heterogeneous wireless networks.

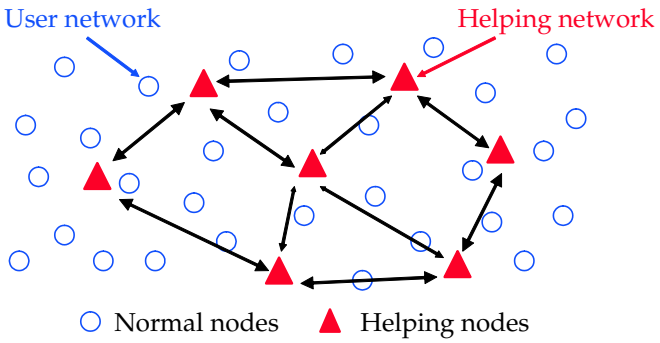


Fig. 1. Our proposed two-tier network model.

A. Network Topology

We consider a dense network with n *normal nodes* and $m = n^b$ $(0 < b < 1)$ more powerful *helping nodes* in a rectangular area with width $b(n)$ and length $1/b(n)$, where $b(n) = n^w$ and $-\frac{1}{2} < w \leq 0$. We assume the n normal nodes are uniformly and independently distributed, while the m helping nodes are either regularly placed or uniformly and independently distributed. The resulting two kinds of networks are called *Regular Heterogeneous Wireless Networks* and *Random Heterogeneous Wireless Networks*, respectively, which will be discussed in Section III and Section IV shortly.

As shown in Fig. 1, our network model has a two-tier hierarchy. Notice that all transmissions in the network are carried out via wireless medium, which is different from hybrid wireless networks.

B. Traffic Pattern

Instead of symmetric traffic mostly assumed in the literature, we assume the network has asymmetric traffic. Specifically, we consider there are n flows in the network. All the n normal nodes are sources while only randomly chosen n^d $(0 < d < 1)$ normal nodes are destinations. Helping nodes do not serve as data sources or data destinations. Instead, they only help relay packets for the normal nodes.

C. Achievable Transmission Rate

Let d_{ij} denote the distance between a node i and another node j. The reception power at node j of the signal from node i, denoted by P_{ij}, follows the power propagation model described in [30], i.e.,

$$P_{ij} = C\frac{P_i}{d_{ij}^{\gamma}}, \tag{1}$$

where P_i is the transmission power of node i, γ is the path loss exponent, and C is a constant related to the antenna profiles of the transmitter and the receiver, wavelength, and so on. As a common assumption, we assume $\gamma > 2$ in outdoor environments [30].

We consider the Shannon Capacity as the achievable transmission rate between two nodes. Specifically, a transmission

from node i to node j can achieve transmission rate, R_{ij}, which is calculated as follows:

$$R_{ij} = W \log(1 + SINR_{ij}), \tag{2}$$

where W is the channel bandwidth, and $SINR_{ij} = \frac{C\frac{P_i}{d_{ij}^{\gamma}}}{N+\sum_{k\neq i} C\frac{P_k}{d_{kj}^{\gamma}}}$ is the SINR (Signal-to-Interference plus Noise Ratio) of the signal from node i to node j. In this paper, we assume the n normal nodes employ the same transmission power $P(n)$ for all their transmissions, and the m helping nodes use the same transmission power $P'(m)$ for the transmissions among themselves.

III. Throughput Capacity of Regular Heterogeneous Wireless Networks

In this section, we derive a lower bound on the throughput capacity of heterogeneous wireless networks by presenting an achievable per-node throughput. We assume the helping nodes are regularly placed. As shown in Fig. 2, transmissions in the network can be carried out either in *user mode* or in *help mode*. In user mode, packets are forwarded from a source node to a destination node with the help of only normal nodes, i.e., without the help of helping nodes. While in help mode, packets are firstly transmitted from a source node to the helping network, and then forwarded to the intended destination user. Besides, we assume all the normal nodes have a total bandwidth of 1, which is split into three frequency bands, i.e., W_1 for ad hoc transmissions in user mode, W_2 for uplink transmissions in help mode, and W_3 for downlink transmissions in help mode, respectively. Thus,

$$W_1 + W_2 + W_3 = 1.$$

We also assume the ad hoc transmissions in help mode have a bandwidth W_4, which may have much higher order than 1 since it is the bandwidth of backbone helping network.

Let T_u and T_h denote an achievable per-node throughput by all nodes when all the transmissions are carried out in user mode and in help mode, respectively. Then, a per-node throughput achievable by all nodes in heterogeneous wireless networks, denoted by T, can be calculated as follows:

$$T = \max\{T_u, T_h\}. \tag{3}$$

In the following, we will derive the throughput in user mode and in help mode, respectively. The basic idea is that given any source-destination pair, if we could find a path between them and the scheduling for all nodes on the path to transmit, then the resulting throughput will be an achievable throughput in this network. Thus, in order to find an achievable throughput in the network, we need to give the medium access control (MAC) and routing strategies first.

A. Achievable Throughput in User Mode

We first introduce the MAC strategy. We divide the network into *squares* with length $l = \sqrt{c_1 \log n / n}$ where $c_1 (c_1 > 1)$ is a constant. Then, we have the following lemma.

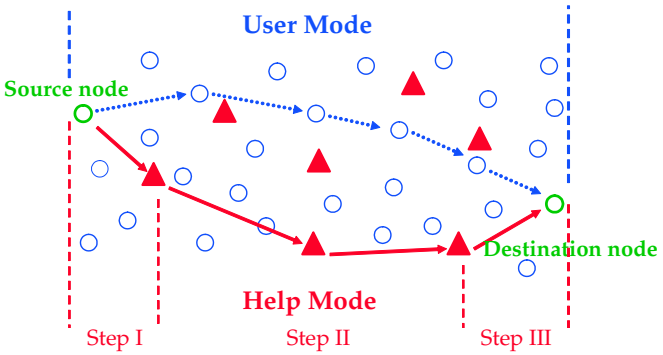


Fig. 2. Transmissions in two modes in the network.

Lemma 1: Every square contains at least one normal node with high probability (w.h.p.).

Proof: For square i, the probability that there is at least one normal node in it, denoted by $\mathbb{P}_i$, as $n \to \infty$, is

$$\mathbb{P}_i = 1 - (1 - l^2)^n = 1 - e^{n \log(1-l^2)} = 1 - \frac{\mathrm{O}(1)}{n^{c_1}}.$$

So, $\mathbb{P}_i \to 1$ as $n \to \infty$. Moreover, let n_s be the number of squares in the network. We have

$$n_s = \frac{1}{l^2} = \frac{n}{c_1 \log n}.$$

Then, the probability that every square has at least one normal node in it, denoted by $\mathbb{P}_A$, is

$$\mathbb{P}_A = \mathbb{P}_i^{n_s} = \left(1 - \frac{\mathrm{O}(1)}{n^{c_1}}\right)^{n_s} = e^{-\frac{\mathrm{O}(1)}{\left(c_1 n^{c_1 - 1}\right) \cdot \log n}}.$$

Since $c_1 > 1$, we obtain that $\mathbb{P}_A \to 1$ as $n \to \infty$, i.e., no square is empty w.h.p.. ■

Besides, in the network, we allow a transmission between two normal nodes only when they are located in two neighboring squares. Notice that each square can have at most four neighboring squares. Thus, we arrive at the following lemma.

Lemma 2: Each square in the network can transmit at a transmission rate $c_2 W_1$ where c_2 is a deterministic positive constant.

Proof: We further divide the network into groups each of which contains nine squares. As shown in Fig. 3, the nine squares in each group are numbered from 1 to 9 in the same way. We also divide time into sequences of successive slots, denoted by t ($t = 0, 1, 2, 3, ...$). During a slot t, all squares that are numbered $(t \mod 9) + 1$ are allowed to transmit packets simultaneously.

Consider a slot when square s_i is allowed to transmit. Then, those squares that may interfere with s_i are located along the perimeters of concentric squares centered at s_i. Since we only allow transmissions between two neighboring squares, at jth tier, there are at most $8j$ interfering squares that are at least $(3j - 2)l$ away from the receiver of s_i. Besides, recall that the network is a rectangle with width $b(n)$ and length $1/b(n)$ where $b(n) = n^w$ and $-1/2 < w \leq 0$. Denote the maximum value of j by J. Obviously, we have $j \leq J < +\infty$. Thus, with the power propagation model in (1), the cumulative

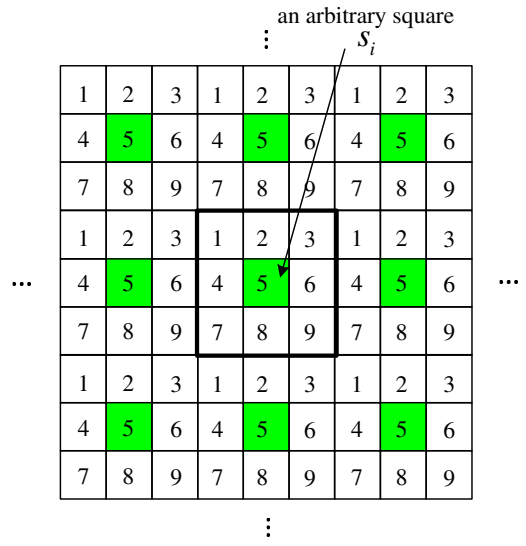


Fig. 3. An example for dividing the network into groups of 9 squares.

interference to the transmission of square s_i, denoted by I_i, can be calculated as

$$\begin{aligned} I_i &\leq \sum_{j=1}^{J} 8j \times \frac{CP(n)}{[(3j-2)l]^\gamma} \\ &\leq \frac{8CP(n)}{l^\gamma}\left[1+\sum_{j=2}^{J}(3j-2)^{(1-\gamma)}\right] \\ &< \frac{8CP(n)}{l^\gamma}\left[1+\int_{j=0}^{+\infty}(3j+1)^{(1-\gamma)}dj\right] \\ &< \frac{8CP(n)}{l^\gamma}\left[1+\frac{1}{3(\gamma-2)}\right] \\ &= \frac{8CP(n)}{l^\gamma}\cdot\frac{3\gamma-5}{3\gamma-6}. \end{aligned} \tag{4}$$

We also need a lower bound on the reception power level at the receiver of s_i, denoted by R_i. Since the maximum distance for a transmitter to a receiver is $\sqrt{5}l$, we can have

$$R_i \geq \frac{CP(n)}{(\sqrt{5}l)^\gamma}. \tag{5}$$

As a result, the SINR at the receiver of s_i, denoted by $SINR_i$, is:

$$\begin{aligned} SINR_i &= \frac{R_i}{N_0+I_i} \\ &\geq \frac{\frac{CP(n)}{(\sqrt{5}l)^\gamma}}{N_0+\frac{8CP}{l^\gamma}\cdot\frac{3\gamma-5}{3\gamma-6}}, \end{aligned}$$

where N_0 is the ambient noise power at the receiver. By choosing the transmission power $P(n) = c_3 l^\gamma$ where c_3 $(0 < c_3 < +\infty)$ is a constant, we can obtain a lower bound on $SINR_i$, i.e.,

$$SINR_i \geq \frac{c_3 C}{5^{\frac{\gamma}{2}}\left(N_0+8c_3C\frac{3\gamma-5}{3\gamma-6}\right)},$$

which is a constant irrespective to the number of nodes n. Thus, referring to (2), we find that in every nine time slots, each square has a chance to transmit at a constant transmission rate. As a result, each square in the network can transmit at a constant transmission rate c_2W_1, where $0 < c_2 < +\infty$. ■

Recall that transmissions in user mode are carried out with only the help of normal nodes. We employ the following routing strategy to relay the packets. Specifically, as shown in Fig. 4(a), assume a source node is located in square s_j and its destination node is located in square s_k. Packets from this source node are firstly relayed along those squares that have the same x-coordinate as square s_j until they arrive at a square that has the same y-coordinate as square s_k. Then, these packets are relayed along the squares that have the same y-coordinate as square s_k until they arrive at the destination node.

Consider an arbitrary square s_i in the network as shown in Fig. 4(b). Let N_x and N_y denote the number of source nodes which are located in squares with the same x-coordinate as s_i, and the number of destination nodes which are located in squares with the same y-coordinate as s_i, respectively. Thus, we have

$$\mathbb{E}[N_x] = n\cdot\frac{l}{1/b(n)} = n^{w+\frac{1}{2}}(c_1\log n)^{\frac{1}{2}}, \tag{6}$$

$$\mathbb{E}[N_y] = n^d\cdot\frac{l}{b(n)} = n^{d-w-\frac{1}{2}}(c_1\log n)^{\frac{1}{2}}. \tag{7}$$

Then, we can obtain the following lemma.

Lemma 3: For every square, w.h.p.,

1) there are at most $2n^{w+\frac{1}{2}}(c_1\log n)^{\frac{1}{2}}$ source nodes which are located in squares with the same x-coordinate.
2) the number of destination nodes which are located in squares with the same y-coordinate is at most $2n^{d-w-\frac{1}{2}}(c_1\log n)^{\frac{1}{2}}$ when $0 < w+\frac{1}{2} < d < 1$, and at most c_4 when $0 < d < w+\frac{1}{2} \leq \frac{1}{2}$, where c_4 is a constant and $c_4 > \frac{2}{w-d+\frac{1}{2}}$.

Proof: Recall the Chernoff bounds [5]:

- For any $\delta > 0$,

$$\mathbb{P}\left(X_i > (1+\delta)\mathbb{E}[X_i]\right) < e^{-\mathbb{E}[X_i]f(\delta)} \tag{8}$$

where $f(\delta) = (1+\delta)\log(1+\delta) - \delta$.

- For any $0 < \delta < 1$,

$$\mathbb{P}(X_i < (1-\delta)\mathbb{E}[X_i]) < e^{-\frac{1}{2}\delta^2\mathbb{E}[X_i]}. \tag{9}$$

1) According to the Chernoff bound in (8), we obtain that

$$\mathbb{P}\left(N_x > 2n^{w+\frac{1}{2}}(c_1\log n)^{\frac{1}{2}}\right) < e^{-f(1)n^{w+\frac{1}{2}}(c_1\log n)^{\frac{1}{2}}},$$

where $f(1) = 2\log 2 - 1 > 0$. Since $-\frac{1}{2} < w \leq 0$, as $n\to\infty$, we have $\mathbb{P}\left(N_x > 2n^{w+\frac{1}{2}}(c_1\log n)^{\frac{1}{2}}\right) \to 0$. Let $\mathbb{P}\left(N_x \leq 2n^{w+\frac{1}{2}}(c_1\log n)^{\frac{1}{2}}\ \forall\ i\right)$ denote the probability that for each square the number of source nodes

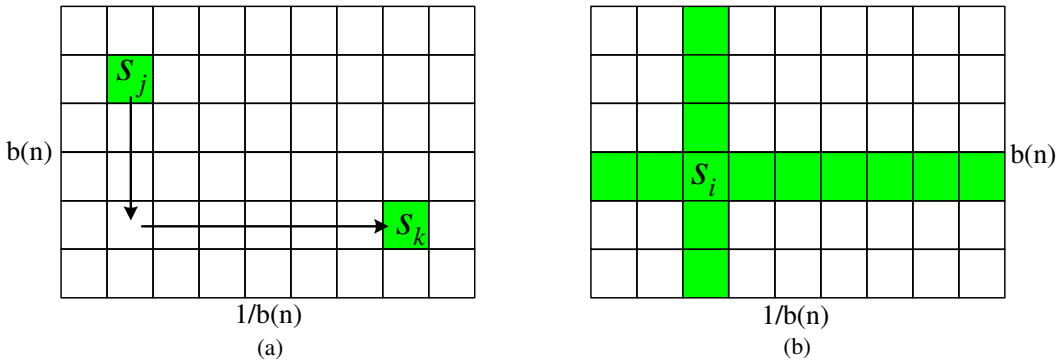


Fig. 4. The routing protocol used for ad hoc mode transmissions.

located in squares with the same x-coordinate is at most $2n^{w+\frac{1}{2}}(c_1 \log n)^{\frac{1}{2}}$. We can obtain that

$$\begin{aligned}&\mathbb{P}\left(N_x \le 2n^{w+\frac{1}{2}}(c_1\log n)^{\frac{1}{2}})\ \forall\ i\right)\\ &\ge 1-\frac{n}{c_1\log n}\mathbb{P}\left(N_x > 2n^{w+\frac{1}{2}}(c_1\log n)^{\frac{1}{2}}\right)\\ &> 1-\frac{n}{c_1\log n}e^{-f(1)n^{w+\frac{1}{2}}(c_1\log n)^{\frac{1}{2}}},\end{aligned}$$

which approaches to 1 as $n\to\infty$.

2) *Firstly*, when $0 < w+\frac{1}{2} < d < 1$, we have

$$\mathbb{P}\left(N_y > 2n^{d-w-\frac{1}{2}}(c_1\log n)^{\frac{1}{2}}\right) < e^{-f(1)n^{d-w-\frac{1}{2}}(c_1\log n)^{\frac{1}{2}}},$$

which approaches to 0 as $n\to\infty$. Similar to that in 1), we can easily show that $\mathbb{P}\left(N_y \le 2n^{d-w-\frac{1}{2}}(c_1\log n)^{\frac{1}{2}}\ \forall\ i\right)\to 1$ as $n\to\infty$.

Secondly, when $0 < d < w+\frac{1}{2} \le \frac{1}{2}$, according to the Chernoff bound in (8), we can obtain that

$$\begin{aligned}\mathbb{P}(N_y > (1+\delta)\mathbb{E}[N_y]) &< e^{-\mathbb{E}[N_y][(1+\delta)\log(1+\delta)-\delta]}\\ &= \frac{e^{\delta\mathbb{E}[N_y]}}{(1+\delta)^{(1+\delta)\mathbb{E}[N_y]}}.\end{aligned}$$

Choose $1+\delta = \frac{c_4}{\mathbb{E}[N_y]} = c_4 n^{w-d+\frac{1}{2}}(c_1\log n)^{-\frac{1}{2}}$ where c_4 is a constant that will be determined later. Then, we have

$$\begin{aligned}\mathbb{P}(N_y > c_4) &< \frac{e^{[c_4 n^{w-d+\frac{1}{2}}(c_1\log n)^{-\frac{1}{2}}-1]n^{d-w-\frac{1}{2}}(c_1\log n)^{\frac{1}{2}}}}{\left[c_4 n^{w-d+\frac{1}{2}}(c_1\log n)^{-\frac{1}{2}}\right]^{c_4}}\\ &= \frac{e^{c_4-n^{d-w-\frac{1}{2}}(c_1\log n)^{\frac{1}{2}}}}{c_4^{c_4}\left[n^{w-d+\frac{1}{2}}(c_1\log n)^{-\frac{1}{2}}\right]^{c_4}}\\ &< \frac{e^{c_4}}{c_4^{c_4}}\cdot\left[n^{d-w-\frac{1}{2}}(c_1\log n)^{\frac{1}{2}}\right]^{c_4},\end{aligned}$$

which approaches to 0 as $n\to\infty$. Besides, we can also obtain that

$$\begin{aligned}\mathbb{P}(N_y \le c_4\ \forall\ i) &\ge 1-\frac{n}{c_1\log n}\mathbb{P}(N_x > c_4)\\ &> 1-\frac{e^{c_4}}{c_4^{c_4}}n^{(d-w-\frac{1}{2})c_4+1}(c_1\log n)^{\frac{1}{2}c_4-1}.\end{aligned}$$

When we choose $c_4 > \frac{2}{w-d+\frac{1}{2}}$, we can get $(d-w+\frac{1}{2})c_4+1 < -1$, and hence $\mathbb{P}(N_y \le c_4\ \forall\ i)\to 1$ as $n\to\infty$.

■

Besides, in the network we have n source nodes while only n^d destination nodes. We can also have a lemma as follows.

Lemma 4: For each destination node, w.h.p., there are at most $2n^{1-d}$ source nodes destined to it.

Proof: Consider destination node i. Let N_i be a random variable denoted as the number of source nodes that have i as their destination node, and $\mathbb{E}[N_i]$ the expectation of N_i. Then, we have $\mathbb{E}[N_i] = n\cdot\frac{1}{n^d} = n^{1-d}$.

According to the Chernoff bound in (8), we can obtain that

$$\mathbb{P}(N_i > 2n^{1-d}) < e^{-f(1)n^{1-d}},$$

where $f(1) > 0$ and $1-d > 0$. So, $\mathbb{P}(N_i > 2n^{1-d})\to 0$ as $n\to\infty$. Besides, we also have

$$\begin{aligned}\mathbb{P}(N_i \le 2n^{1-d}\ \forall\ i) &\ge 1-n^d\mathbb{P}(N_i > 2n^{1-d})\\ &> 1-n^d e^{-f(1)n^{1-d}},\end{aligned}$$

which approaches to 1 as $n\to\infty$. ■

Denote the number of flows that cross square s_i as F_i. Since each source node only generates one flow, and there are at most $2n^{1-d}$ flows for each destination node as shown in Lemma 4, we can obtain that for all i,

$$\begin{aligned}F_i &\le N_x + 2n^{1-d}N_y\\ &\le \begin{cases}2n^{w+\frac{1}{2}}(c_1\log n)^{\frac{1}{2}} + 4n^{\frac{1}{2}-w}(c_1\log n)^{\frac{1}{2}},\\ \qquad\qquad \text{when } 0 < w+\frac{1}{2} < d < 1,\\ 2n^{w+\frac{1}{2}}(c_1\log n)^{\frac{1}{2}} + 2c_4 n^{1-d},\\ \qquad\qquad \text{when } 0 < d < w+\frac{1}{2} \le \frac{1}{2},\end{cases}\end{aligned}$$

i.e.,

$$F_i = \mathrm{O}\left(\max\left\{n^{w+\frac{1}{2}}(\log n)^{\frac{1}{2}}, n^{\frac{1}{2}-w}(\log n)^{\frac{1}{2}}, n^{1-d}\right\}\right). \tag{10}$$

Recall that we have proved in Lemma 2 that a constant transmission rate $c_2 W_1$ can be achieved for each transmission. Thus, from (10), we can obtain

$$T_u = \Omega\left(\min\left\{\frac{n^{-w-\frac{1}{2}}}{\sqrt{\log n}}W_1, \frac{n^{w-\frac{1}{2}}}{\sqrt{\log n}}W_1, n^{d-1}W_1\right\}\right).$$

Since $-\frac{1}{2} < w \leq 0$, we get $-w - \frac{1}{2} \geq -\frac{1}{2}$ and $w - \frac{1}{2} \leq -\frac{1}{2}$. So, we obtain that

$$T_u = \Omega\left(\min\left\{\frac{n^{w-\frac{1}{2}}}{\sqrt{\log n}}, n^{d-1}\right\}\right). \tag{11}$$

B. Achievable Throughput in Help Mode

Transmissions in help mode are carried out in three steps: from a source node to the nearest helping node to it, from this helping node to the helping node nearest to the destination node, and finally from that helping node to the destination node. We analyze the throughput capacity in these three steps, respectively, in the following, and the minimum of these three is an achievable throughput in help mode.

Step I: from source nodes to the helping network.

Recall that in regular heterogeneous wireless networks the helping nodes are regularly placed. So a network can be divided into squares of equal area $1/m$, which we call *cells*. We assume source nodes have low transmission power and they need to transmit packets to the helping node in the same cell via multiple hops. Then, we can have the following lemma. The proof is omitted due to space limit.

Lemma 5: In each cell, there are at most $2n/m$ normal nodes w.h.p., where $m = n^b$, the number of helping nodes.

In Lemma 1, we have shown that in each square, there exists at least one normal node that can help relay traffic. Recall that a bandwidth of W_2 is used for the transmissions in Step I. So similar to Lemma 2, we can also show that each square can transmit at a constant transmission rate $c_6 W_2$ where $0 < c_6 < +\infty$ is a deterministic constant. Besides, by Lemma 5, in each cell, there are at most $\frac{2n}{m} - 1$ normal nodes that one square has to relay traffic for. Assume each packet has a constant packet size. So, in $\mathrm{O}\left(9 \times \frac{2n}{m}\right)$ time slots, every normal node is able to transmit a packet towards the helping node closest to it.

Denote the throughput capacity in Step I by T_{h1}. We can obtain that

$$T_{h1} = \Omega\left(\frac{m}{n} W_2\right) = \Omega\left(n^{b-1}\right). \tag{12}$$

Step II: helping network relay.

Notice that each cell is a *big square* with length $l' = \sqrt{1/m}$, and there is exactly one helping node in each cell. Thus, similar to Lemma 2, we can also have the following lemma.

Lemma 6: Each cell in the network can transmit at a transmission rate $c_7 W_4$, where c_7 is a deterministic positive constant.

Besides, as shown in Fig. 5, notice that the m big squares with length $l' = \sqrt{1/m}$ cannot cover the whole network when $b(n) = \mathrm{O}(l')$, i.e., $w + b/2 < 0$. In this case, we choose $l' = n^{-w-b}$ to ensure full network coverage. As a result, we have the following lemma.

Lemma 7: For every cell, w.h.p.,

1) the number of source nodes which are located in the cells with the same x-coordinate, denoted by C_x, is at most $2n^{w-\frac{b}{2}+1}$ when $w + \frac{b}{2} > 0$, and at most $2n^{1-b}$ when $w + \frac{b}{2} < 0$.
2) the number of destination nodes which are located in the cells with the same y-coordinate, denoted by C_y, is at most $2n^{d-w-\frac{b}{2}}$ when $d > w + \frac{b}{2} > 0$, at most c_8 when $w + \frac{b}{2} > d > 0$, where c_8 is a constant and $c_8 > \frac{4}{b+2w-2d}$, and at most n^d when $w + \frac{b}{2} < 0$.

Proof: When $w + \frac{b}{2} > 0$, the proof is similar to that in Lemma 3. When $w + \frac{b}{2} < 0$, there is only one cell for each x-coordinate, and hence the number of source nodes is the same as shown in Lemma 5. Besides, all the cells have the same y-coordinate. So the number of destination nodes located in the cells with the same y-coordinate is at most the total number of destination nodes in the network, i.e., n^d. ■

Denote the number of flows that cross an arbitrary cell C_i as B_i where $i \in [1, n]$. Remember that each source node only generates one flow, and there are at most $2n^{1-d}$ flows for each destination node as shown in Lemma 4, we can obtain that for all i,

$$\begin{aligned} B_i &\leq C_x + 2n^{1-d} C_y \\ &\leq \begin{cases} 2n^{w-\frac{b}{2}+1} + 4n^{-w-\frac{b}{2}+1}, & \text{when } 0 < w + \frac{b}{2} < d < 1, \\ 2n^{w-\frac{b}{2}+1} + 2c_3 n^{1-d}, & \text{when } 0 < d < w + \frac{b}{2} < 1, \\ 2n^{1-b} + n, & \text{when } w + \frac{b}{2} < 0. \end{cases} \end{aligned}$$

Denote the throughput capacity in Step II by T_{h2}. Thus, from Lemma 6, we can obtain that

$$\begin{aligned} T_{h2} &= \Omega\left(\min\left\{n^{-w+\frac{b}{2}-1} W_4, n^{I(w+\frac{b}{2})-1} W_4, n^{d-1} W_4\right\}\right) \\ &= \Omega\left(\min\left\{n^{I(w+\frac{b}{2})-1} W_4, n^{d-1} W_4\right\}\right), \end{aligned} \tag{13}$$

where $I(.)$ is a function, and $I(x) = x$ if $x > 0$, and $I(x) = 0$ if $x < 0$.

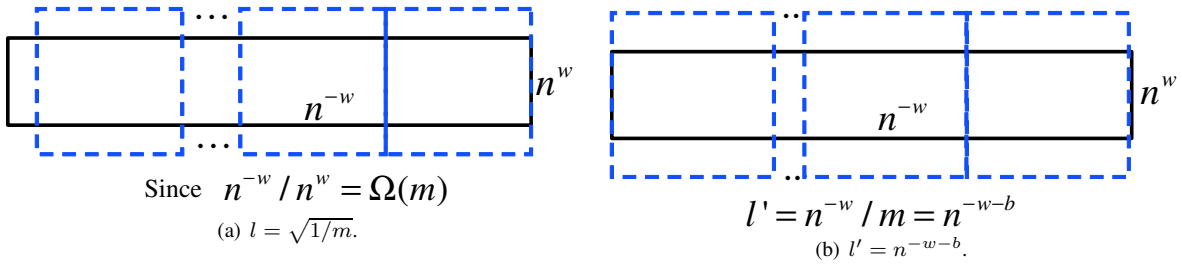
Step III: from the helping network to destination nodes.

We first give two lemmas that will be used later.

Lemma 8: In each cell, w.h.p., there are at most $2n^{d-b}$ destination nodes when $0 < b < d < 1$, and at most c_9, where $c_9 > \frac{1+b}{b-d}$, destination nodes when $0 < d < b < 1$.

Proof: Consider cell i. Let Y_i be a random variable denoted as the number of destination nodes in cell i, and $\mathbb{E}[Y_i]$ the expectation of Y_i. Then, we have $\mathbb{E}[Y_i] = \frac{n^d}{m} = n^{d-b}$.

1) $0 < b < d < 1$.
   According to the Chernoff bound, we can obtain that
   $$\mathbb{P}\left(Y_i > 2n^{d-b}\right) < e^{-c_{10} n^{d-b}},$$
   where $c_{10} = f(1) = 2\log 2 - 1 > 0$. Thus, as $n \to \infty$, we have $\mathbb{P}\left(Y_i > 2n^{d-b}\right) \to 0$. Besides, the probability that the number of destination nodes is at most $2n^{d-b}$ in all cells, denoted by $\mathbb{P}\left(Y_i \leq 2n^{d-b}\ \forall\ i\right)$, can be calculated as:
   $$\begin{aligned} \mathbb{P}\left(Y_i \leq 2n^{d-b}\ \forall\ i\right) &\geq 1 - m\mathbb{P}\left(Y_i > 2n^{d-b}\right) \\ &> 1 - n^b e^{-c_{10} n^{d-b}}, \end{aligned}$$
   which approaches to 1 as $n \to \infty$.
2) $0 < d < b < 1$.



(a) $l=\sqrt{1/m}$. (b) $l'=n^{-w-b}$.

Fig. 5. A special case for covering the network with m squares.

Again, according to the Chernoff bound introduced before, we can obtain that

$$\begin{aligned}\mathbb{P}(Y_i>(1+\delta)\mathbb{E}[Y_i]) &< e^{-\mathbb{E}[Y_i][(1+\delta)\log(1+\delta)-\delta]}\\ &= \frac{e^{\delta\mathbb{E}[Y_i]}}{(1+\delta)^{(1+\delta)\mathbb{E}[Y_i]}}.\end{aligned}$$

Let $1+\delta=\frac{c_9}{\mathbb{E}[Y_i]}=c_9n^{b-d}$ where c_3 is a constant that will be determined later. Then, we have

$$\begin{aligned}\mathbb{P}(Y_i>c_9) &< \frac{e^{(c_9n^{b-d}-1)n^{d-b}}}{(c_9n^{b-d})^{c_9}}\\ &= \frac{e^{c_9-n^{d-b}}}{c_9^{c_9}n^{c_9(b-d)}}\\ &< \frac{e^{c_9}}{c_9^{c_9}}\cdot n^{c_9(d-b)}\to 0 \text{ as } n\to\infty.\end{aligned}$$

Besides, we can also obtain that

$$\begin{aligned}\mathbb{P}(Y_i\le c_9\ \forall\ i) &\ge 1-m\mathbb{P}(Y_i>c_9)\\ &> 1-n^b\frac{e^{c_9}}{c_9^{c_9}}\cdot n^{c_9(d-b)}\\ &= 1-\frac{e^{c_9}}{c_9^{c_9}}\cdot n^{c_9d-(c_9-1)b}.\end{aligned}$$

When we choose $c_9>\frac{1+b}{b-d}$, we can get $c_9d-(c_9-1)b<-1$, and hence $\mathbb{P}(Y_i\le c_9\ \forall\ i)\to 1$ as $n\to\infty$. ■

Since all the cells are of the same size, we assume the transmission power of helping nodes for the transmission in Step III is strong enough so that helping nodes can directly transmit to destination nodes within the cells. Besides, as in cellular systems, we use 7-cell frequency reuse to enable adjacent cells to transmit at the same time with no interference. Then, it is easy to show that downlink transmissions in Step III can also have a constant transmission rate $c_{11}W_3$ where c_{11} $(0<c_{11}<+\infty)$ is a deterministic constant. In addition, from Lemma 4 and Lemma 8, we find that when $0<b<d<1$, in each cell, w.h.p., the number of flows from a helping node to destination nodes is at most $2n^{d-b}\times 2n^{1-d}$, i.e., $4n^{1-b}$, and when $0<d<b<1$, in each cell, w.h.p., the number of flows from a helping node to destination nodes is at most $c_9\times 2n^{1-d}$, i.e., $2c_9n^{1-d}$. Denote the throughput capacity in Step III by T_{h3}. Then, we can obtain that

$$T_{h3}=\begin{cases}\Omega\left(n^{b-1}W_3\right), & \text{when } 0<b<d<1,\\ \Omega\left(n^{d-1}W_3\right), & \text{when } 0<d<b<1.\end{cases}\quad(14)$$

Thus, combining (12), (13), and (14), we can get

$$\begin{aligned}T_h &= \Omega(\min\{T_{h1},T_{h2},T_{h3}\})\\ &= \Omega\left(\min\left\{n^{b-1},n^{I(w+\frac{b}{2})-1}W_4,n^{d-1}\right\}\right).\end{aligned}\quad(15)$$

C. An Achievable Throughput in Regular Heterogeneous Wireless Networks

Substituting the results in (11) and (15) into (3), we can have the following theorem.

***Theorem** 1:* An achievable throughput in regular heterogeneous wireless networks, denoted by T, is

$$T=\Omega\left(\max\left\{\min\left\{\frac{n^{w-\frac{1}{2}}}{\sqrt{\log n}},n^{d-1}\right\},\ \min\left\{n^{b-1},n^{I(w+\frac{b}{2})-1}W_4,n^{d-1}\right\}\right\}\right).$$

IV. Throughput Capacity of Random Heterogeneous Wireless Networks

In this section, we study the throughput capacity of random heterogeneous wireless networks with uniformly and independently placed helping nodes. Similarly, we also need to find an achievable throughput in user mode and in help mode, denoted by T'_u and T'_h, respectively. Then, an achievable throughput in random heterogeneous wireless networks, denoted by T', can be obtained by choosing the maximum of these two, i.e.,

$$T'=\max\{T'_u,T'_h\}.\quad(16)$$

Notice that T'_u is the same as T_u shown in (11), but T'_h will be different from T_h due to random distribution of the helping nodes. In the following, we present how to find T'_h.

Recall the definition of *Voronoi Tessellation*: given a set of m points in a plane, Voronoi tessellation divides the domain in a set of polygonal regions, the boundaries of which are the perpendicular bisectors of the lines joining the points. It has been shown in [11] (Lemma 4.1) that for every $\varepsilon>0$, there is a Voronoi tessellation with the property that every Voronoi cell contains a disk of radius ε and is contained in a disk of radius 2ε. Then, for the m base stations in a dense network with area 1, we can construct a Voronoi tessellation V_n for which

- (V1) Every Voronoi cell contains a disk of area $\frac{100\log m}{m}$.

- (V2) Every Voronoi cell is contained in a disk of radius $2\rho(n)$, where $\rho(n)$:= the radius of a disk of area $\frac{100\log m}{m}$.

In this case, we consider each voronoi cell is a cell in the network. In each cell, we randomly choose one helping node as the helping node of the cell.

Similar to that in Section III-B, the transmissions in help mode are also carried out in three steps as follows.

Step I: from source nodes to the helping network.

We also assume source nodes have low transmission power and they need to transmit packets to helping nodes via multiple hops. Similar to Lemma 5, we can have the following result.

Lemma 9: In each Voronoi cell, there are at most $\frac{1200n\log m}{m}$ normal nodes w.h.p..

Denote the throughput capacity in Step I by T'_{h1}. Since each node is a source node, along the line in Section III-B, we can obtain that

$$T'_{h1} = \Omega\left(\frac{m}{n\log m}W_2\right) = \Omega\left(\frac{n^{b-1}}{\log n}\right). \tag{17}$$

Step II: helping network relay.

When $w+\frac{b}{2}>0$, we further divide the network into *big squares* with length $l''=\sqrt{c_{12}\log m/m}$ where $c_{12}>1/b$ is a constant. When $w+\frac{b}{2}<0$, we make $l''=c_{12}n^{-w-b}\log m$. Then, we can have the following lemma.

Lemma 10: In random heterogeneous wireless networks,

1) Every big square has at least one helping node in it w.h.p..
2) Each big square in the network can transmit at a constant transmission rate $c_{13}W_2$, where $0<c_{13}<+\infty$ is a deterministic constant.

Besides, similar to Lemma 7, we can have the following lemma.

Lemma 11: For every big square, w.h.p.,

1) the number of source nodes from which the traffic goes through the big square, denoted by C'_x, is $\mathrm{O}\left(n^{w-\frac{b}{2}+1}(\log n)^{\frac{1}{2}}\right)$ when $w+\frac{b}{2}>0$, and $\mathrm{O}(n^{1-b}\log n)$ when $w+\frac{b}{2}<0$.
2) the number of destination nodes to which the traffic goes through the big square, denoted by C'_y, is $\mathrm{O}\left(n^{d-w-\frac{b}{2}}(\log n)^{\frac{1}{2}}\right)$ when $d>w+\frac{b}{2}>0$, at most c_{14} when $w+\frac{b}{2}>d>0$, where c_{14} is a constant and $c_{14}>\frac{4}{b+2w-2d}$, and at most n^d when $w+\frac{b}{2}<0$.

Proof: Notice that in random heterogeneous wireless networks, the expectation of C'_x and C'_y cannot be directly calculated as in (6) and (7), respectively. Instead, for a big square, the expectation of C'_x should be the average number of helping nodes in big squares with the same x-coordinate times the average number of source nodes associated with each big square. The expectation of C'_y should be derived similarly. Thus, we have

$$\mathbb{E}[C'_x] = \Theta\left(\frac{m}{\log m}\right)\cdot\frac{l''}{1/b(n)}\cdot\Theta\left(\frac{n\log m}{m}\right) = \Theta\big(nl''b(n)\big),$$

$$\mathbb{E}[C'_y] = \Theta\left(\frac{m}{\log m}\right)\cdot\frac{l''}{b(n)}\cdot\Theta\left(\frac{n^d\log m}{m}\right) = \Theta\left(\frac{n^d l''}{b(n)}\right).$$

The rest of the proof follows that in Lemma 7. ■

Denote the number of flows that crosses an arbitrary big square S_i as B'_i where $i\in[1,n]$. We can obtain that for all i,

$$\begin{aligned} B'_i &\le C'_x + 2n^{1-d}C'_y \\ &= \begin{cases} \mathrm{O}\left(n^{w-\frac{b}{2}+1}(\log n)^{\frac{1}{2}}\right)+\mathrm{O}\left(n^{-w-\frac{b}{2}+1}(\log n)^{\frac{1}{2}}\right), \\ \qquad\qquad \text{when } d>w+\frac{b}{2}>0, \\ \mathrm{O}\left(n^{w-\frac{b}{2}+1}(\log n)^{\frac{1}{2}}\right)+\mathrm{O}\left(n^{1-d}\right), \\ \qquad\qquad \text{when } w+\frac{b}{2}>d>0, \\ \mathrm{O}\left(n^{1-b}\log n\right)+\mathrm{O}\left(n\right), \text{ when } w+\frac{b}{2}<0. \end{cases} \end{aligned}$$

Denote the throughput capacity in Step II by T'_{h2}. Thus, we can obtain that

$$\begin{aligned} T'_{h2} &= \min\left\{\frac{n^{-w+\frac{b}{2}-1}}{(\log n)^{\frac{1}{2}}}W_4, \frac{n^{I(w+\frac{b}{2})-1}}{(\log n)^{J(w+\frac{b}{2})}}W_4, n^{d-1}W_4\right\} \\ &= \min\left\{\frac{n^{I(w+\frac{b}{2})-1}}{(\log n)^{J(w+\frac{b}{2})}}W_4, n^{d-1}W_4\right\}, \end{aligned} \tag{18}$$

where $J(\cdot)$ is a function, and $J(x)=\frac{1}{2}$ when $x>0$, and $J(x)=0$ when $x<0$.

Step III: from the helping network to destination nodes.

Since the cells are irregular in random heterogeneous wireless networks, we cannot assume the transmissions in Step III can be one hop direct transmissions as in regular heterogeneous wireless networks. Instead, we assume the transmission power of helping nodes for the transmissions in Step III is the same as normal nodes. Then, we can also show that transmissions in Step III can have a constant transmission rate $c_{15}W_3$ where c_{15} $(0<c_{15}<+\infty)$ is a deterministic constant. Besides, similar to Lemma 8, we can have the following lemma.

Lemma 12: In each cell, w.h.p., the number of destination nodes is $\mathrm{O}(n^{d-b}\log n)$ when $0<b<d<1$, and $\mathrm{O}(c_9)$, where $c_9>\frac{1+b}{b-d}$ when $0<d<b<1$.

Denote the throughput capacity in Step III by T'_{h3}. Recall Lemma 4, and we can obtain that

$$T'_{h3} = \begin{cases} \Omega(n^{b-1}W_3/\log n), \text{ when } 0<b<d<1, \\ \Omega(n^{d-1}W_3), \text{ when } 0<d<b<1. \end{cases} \tag{19}$$

Thus, combining (17), (18), and (19), we can obtain

$$\begin{aligned} T'_h &= \Omega(\min\{T'_{h1}, T'_{h2}, T'_{h3}\}) \\ &= \Omega\left(\min\left\{\frac{n^{b-1}}{\log n}, \frac{n^{I(w+\frac{b}{2})-1}}{(\log n)^{J(w+\frac{b}{2})}}W_4, n^{d-1}\right\}\right). \end{aligned} \tag{20}$$

Substituting the results in (20) into (16), we can have the following theorem.

Theorem 2: An achievable throughput in random heterogeneous wireless networks, denoted by T', is

$$\begin{aligned} T' = \Omega\Bigg(\max\Bigg\{&\min\left\{\frac{n^{w-\frac{1}{2}}}{\sqrt{\log n}}, n^{d-1}\right\}, \\ &\min\left\{\frac{n^{b-1}}{\log n}, \frac{n^{I(w+\frac{b}{2})-1}}{(\log n)^{J(w+\frac{b}{2})}}W_4, n^{d-1}\right\}\Bigg\}\Bigg). \end{aligned}$$

From Theorem 1 and Theorem 2, we can find that the number of destination nodes, the number of helping nodes, and the shape of the network area all have significant impacts on the achievable throughput in heterogeneous wireless networks. More importantly, notice that the user mode achievable throughput is in fact the throughput in traditional static homogeneous ad hoc networks. Thus, we can find that instead of placing base stations with a wired network, we can have higher throughput than homogeneous ad hoc networks by an exponential order if we deploy some more powerful wireless nodes with *1)* $W_4 = \Omega(n^{w+\frac{1}{2}})$ when $w + b/2 < 0$ and $w - \min\{b, d\} + 1/2 < 0$, or *2)* $W_4 = \Omega(n^{\frac{1-b}{2}})$ when $w + b/2 > 0$ and $w - \min\{b, d\} + 1/2 < 0$.

V. Conclusion

In this paper, we investigate the throughput capacity of regular and random heterogeneous wireless networks, respectively, and find the network settings such as the number of destination nodes, the number of helping nodes, and the shape of the network area all have great impacts on the network capacity. We also find that by deploying wireless helping nodes into the network, heterogeneous wireless networks can provide much higher per-node throughput than traditional homogeneous wireless networks under certain conditions.

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